A perspective-neutral approach to quantum general covariance

It from Qubit
Simons Collaboration on
Quantum Fields, Gravity and Information

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University College London
and
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OIST

QISS Workshop Hong Kong
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Vanrietvelde, PH, Giacomini, Castro-Ruiz 1809.00556
Vanrietvelde, PH, Giacomini 1809.05093
PH, Vanrietvelde 1810.04153
PH 1811.00611
PH, Smith, Lock 1912.00033

General covariance

"All the laws of physics are the same in <u>all</u> reference frames."

General covariance

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laws as tensor equations

reference-frame-neutral

General covariance

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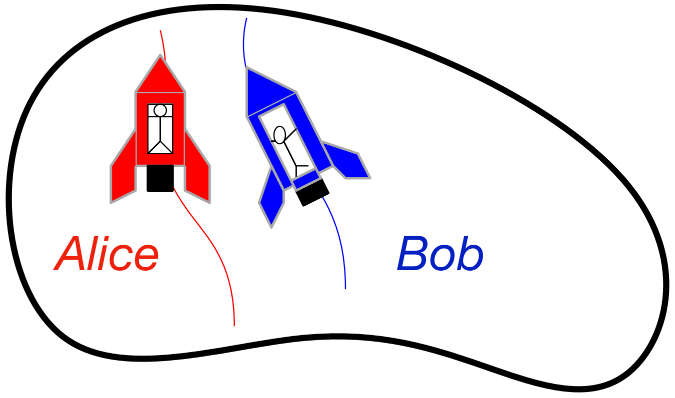
frames usually

- **idealized**
- **external**



spacetime

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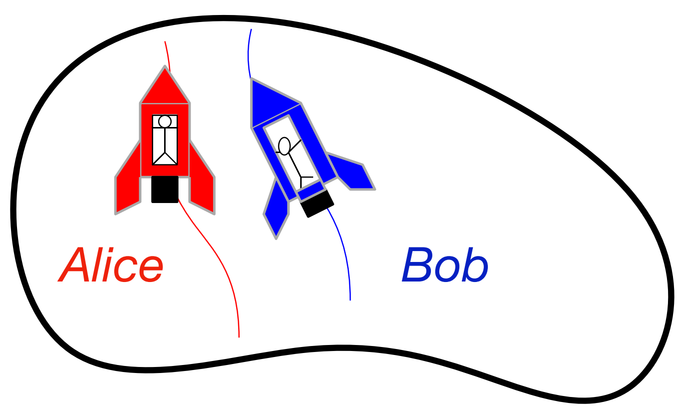
frame-neutral



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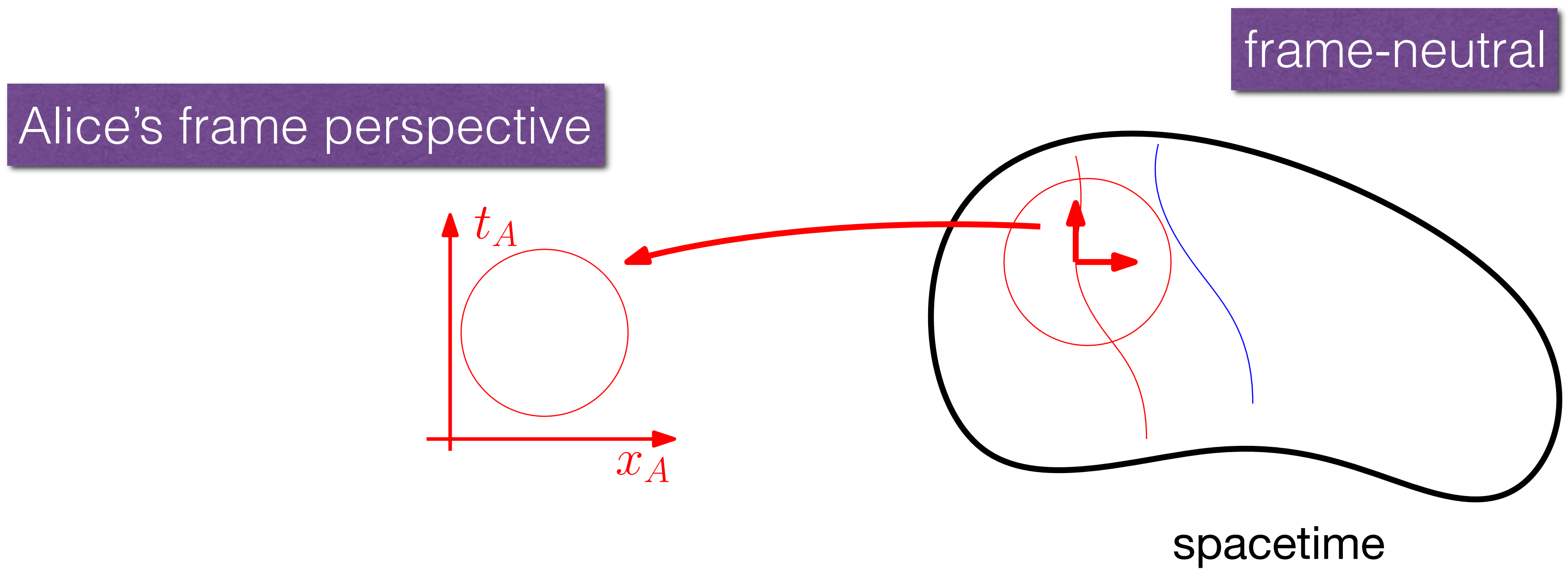

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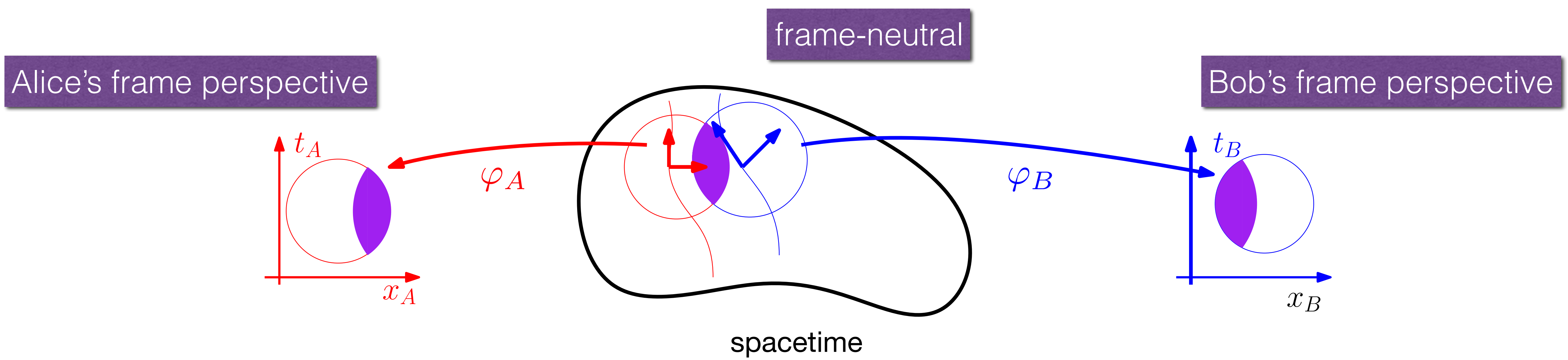

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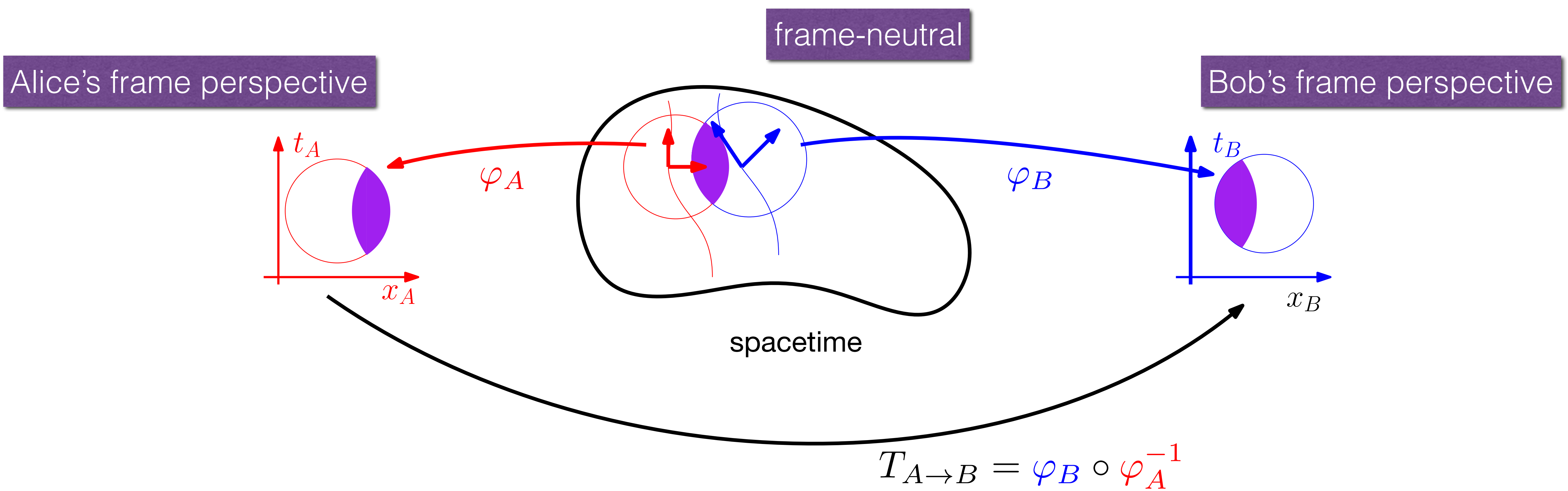
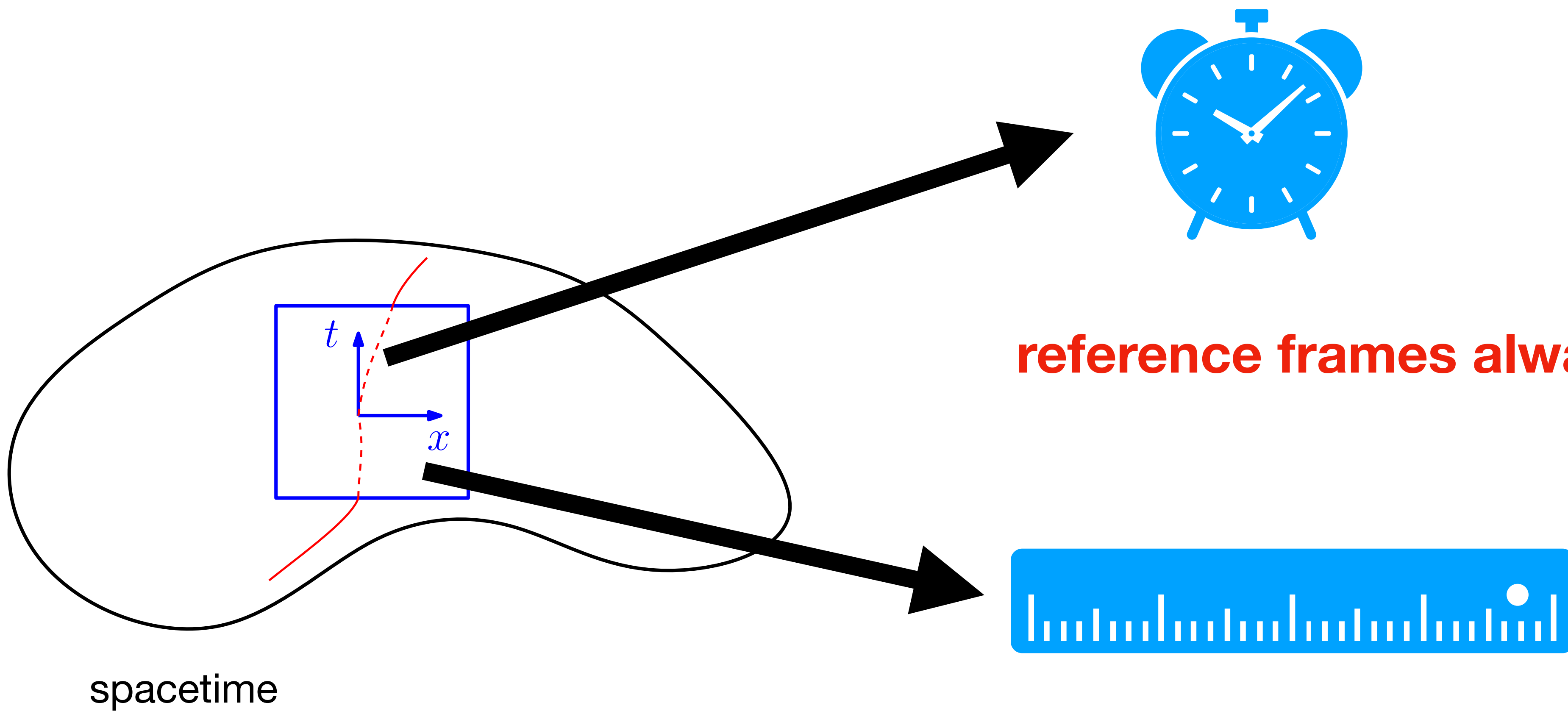


Quantum reference frames/systems

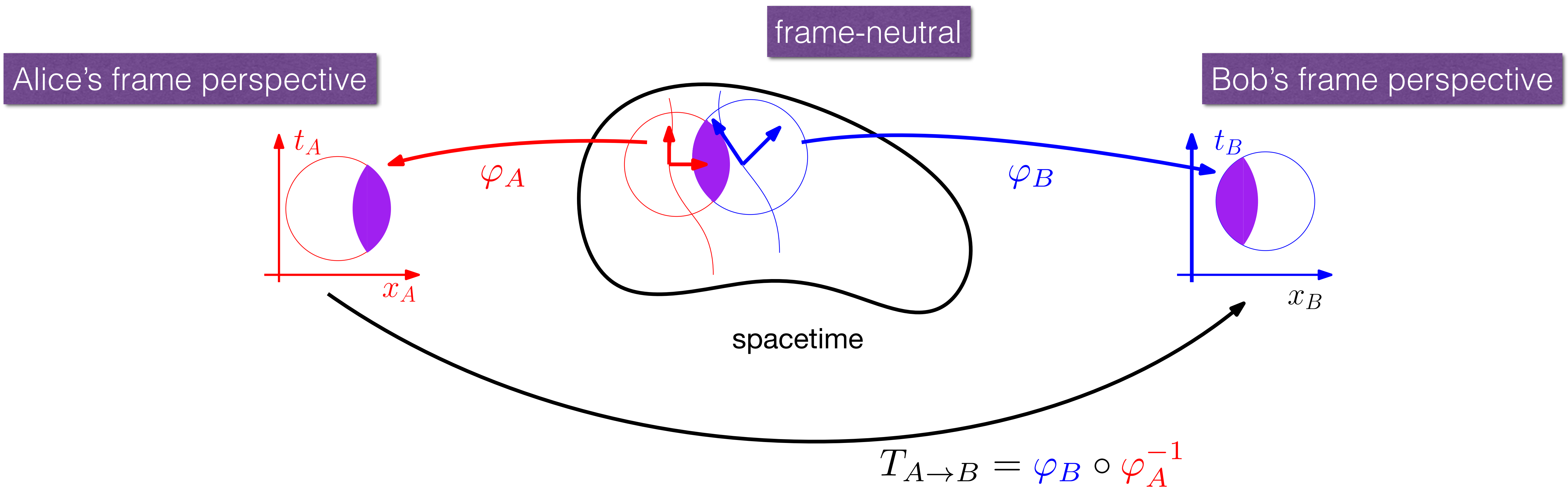


reference frames always physical systems

Universality of QT $\Rightarrow$ RF subject to QM itself

Towards quantum general covariance

“All the laws of physics are the same in <u>all</u> reference frames.”



How to make sense of general covariance when frames are quantum?

Towards quantum general covariance

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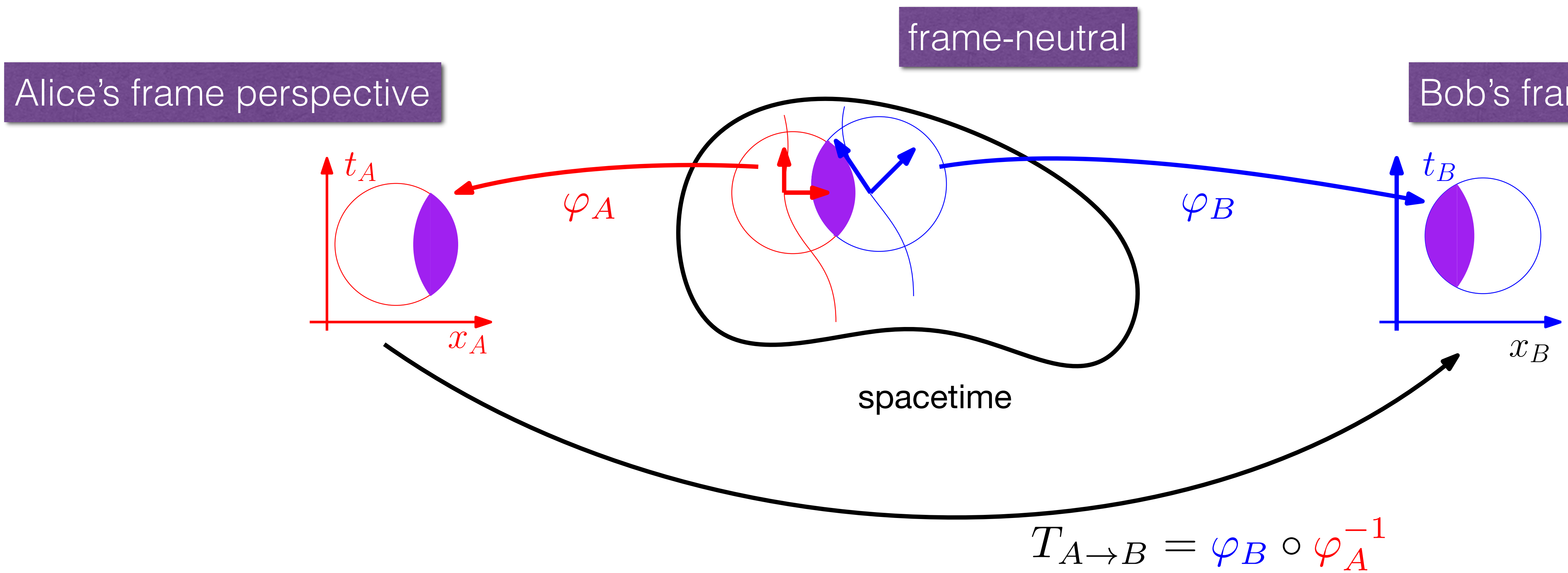

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<u>pre-history:</u>

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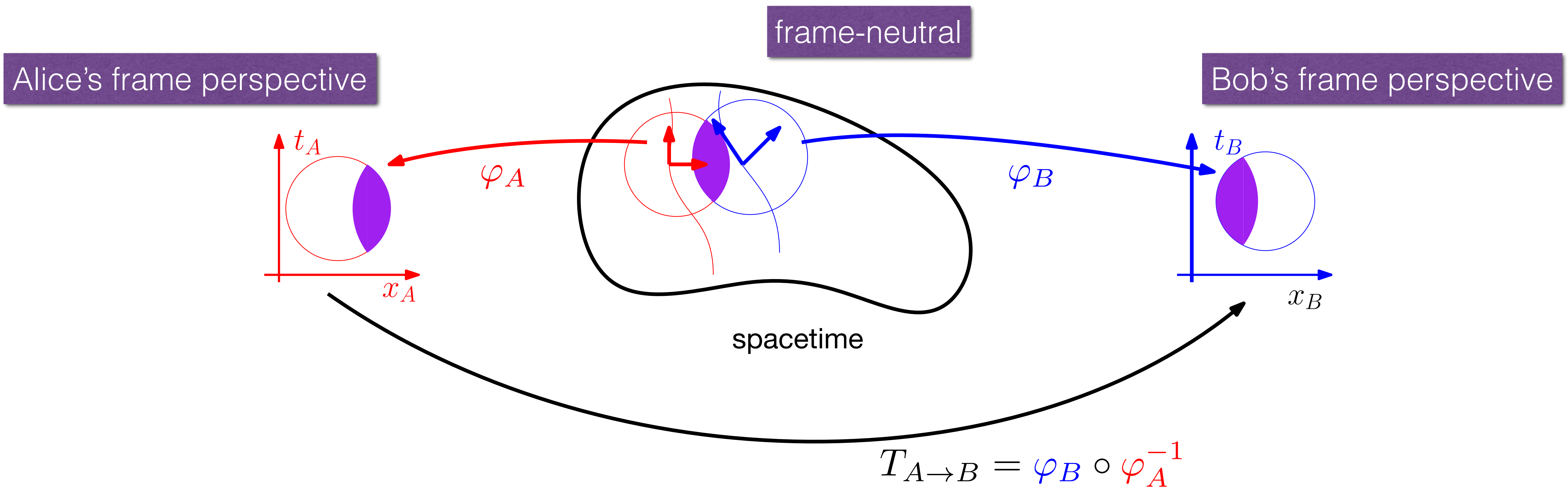

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$\Rightarrow$

<u>perspective-neutral approach:</u>

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Strategy

shift focus from passive to active diffeos

$\Longrightarrow$ get away from coordinate description, focus on dynamical DoFs

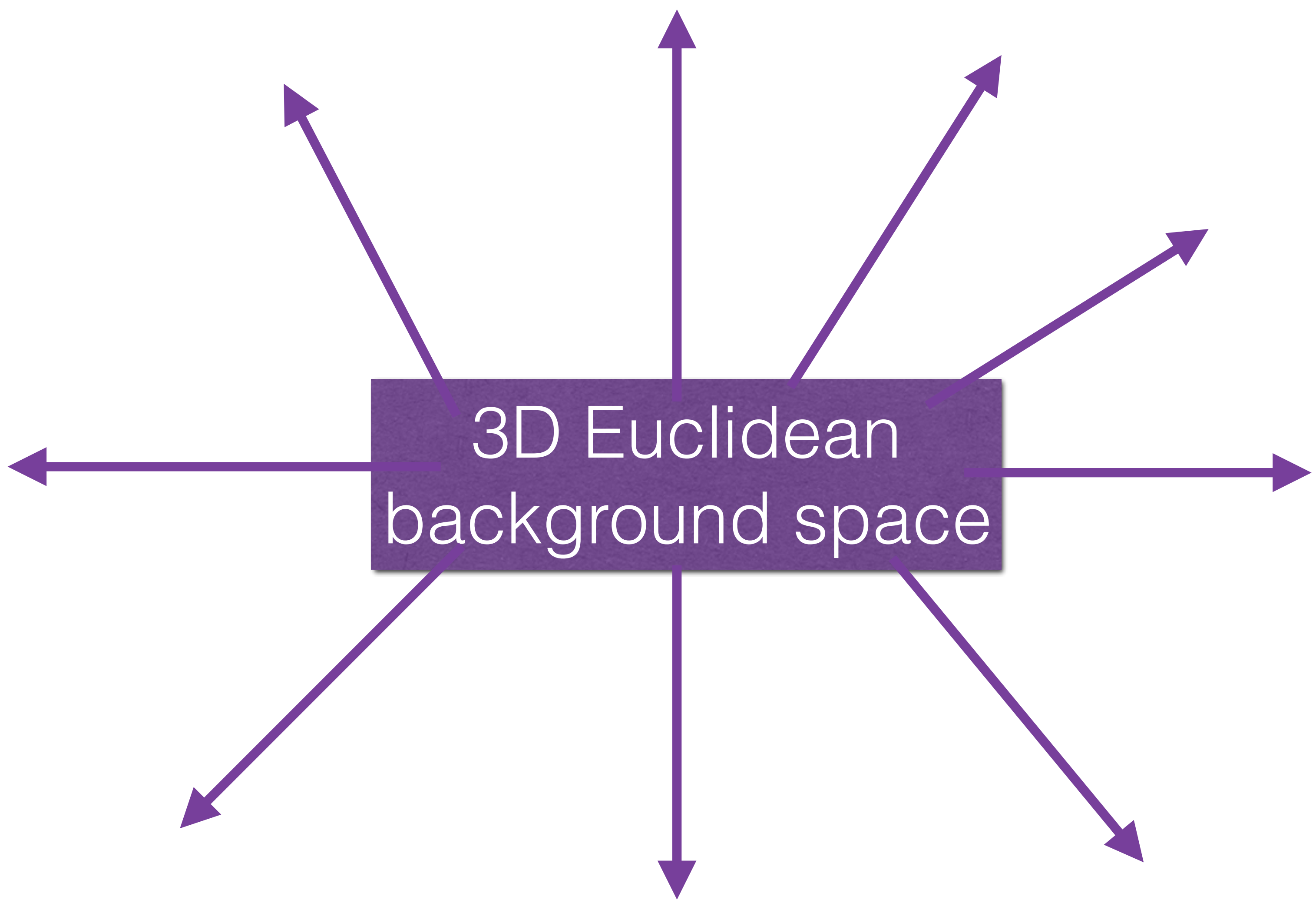
better for QG, were coordinates a priori absent

key: symmetries $\Longrightarrow$ redundancy

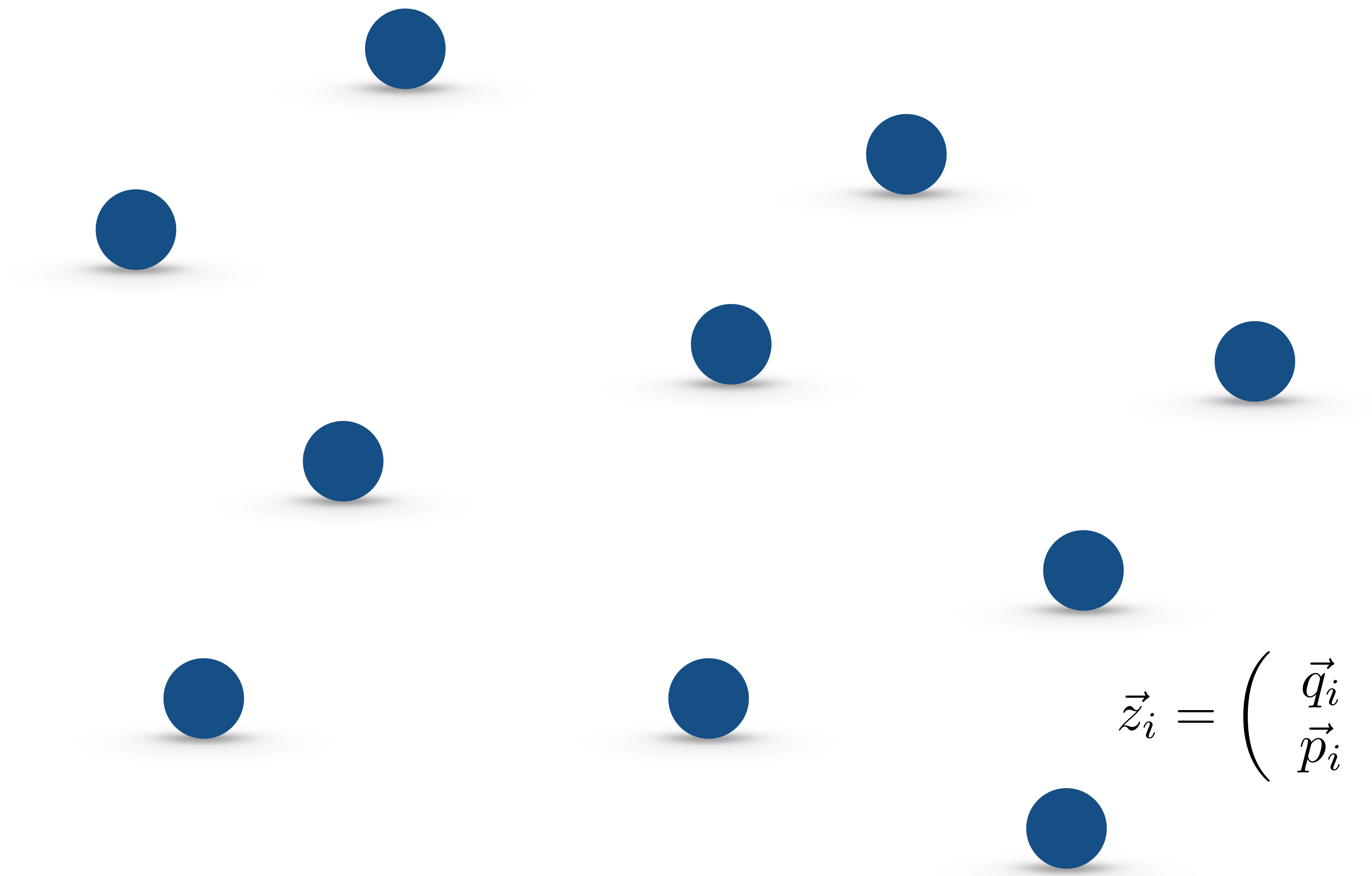
Plan

1. Illustration in relational N-body problem
2. Trinity of relational quantum dynamics
3. Changing quantum clocks
4. Conclusions

The relational N-body problem



The relational N-body problem

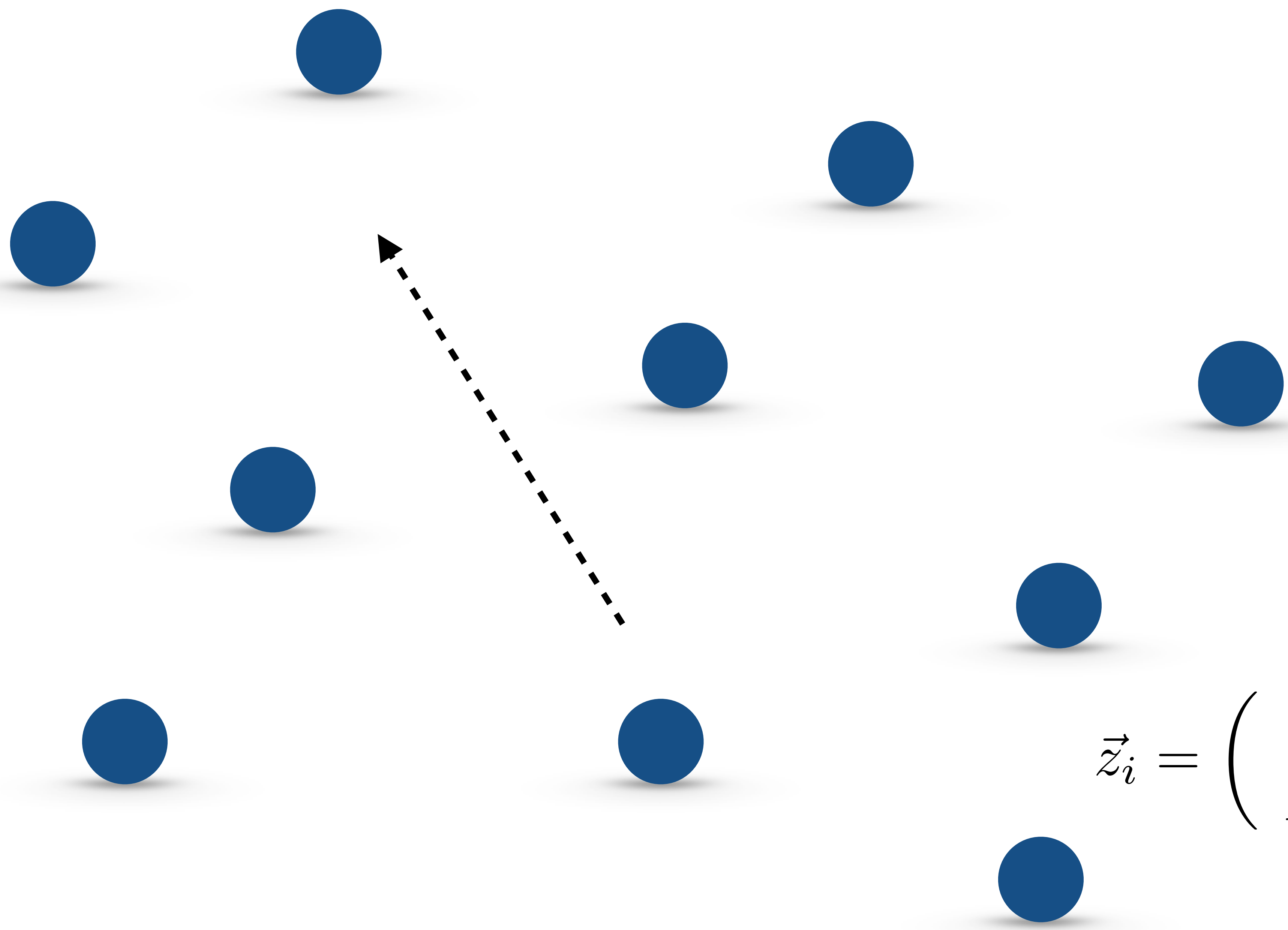


$$\vec{z}_i = \begin{pmatrix} \vec{q}_i \\ \vec{p}_i \end{pmatrix}, \quad i = 1, \ldots, N$$

The relational N-body problem

require invariance under:

1. translations



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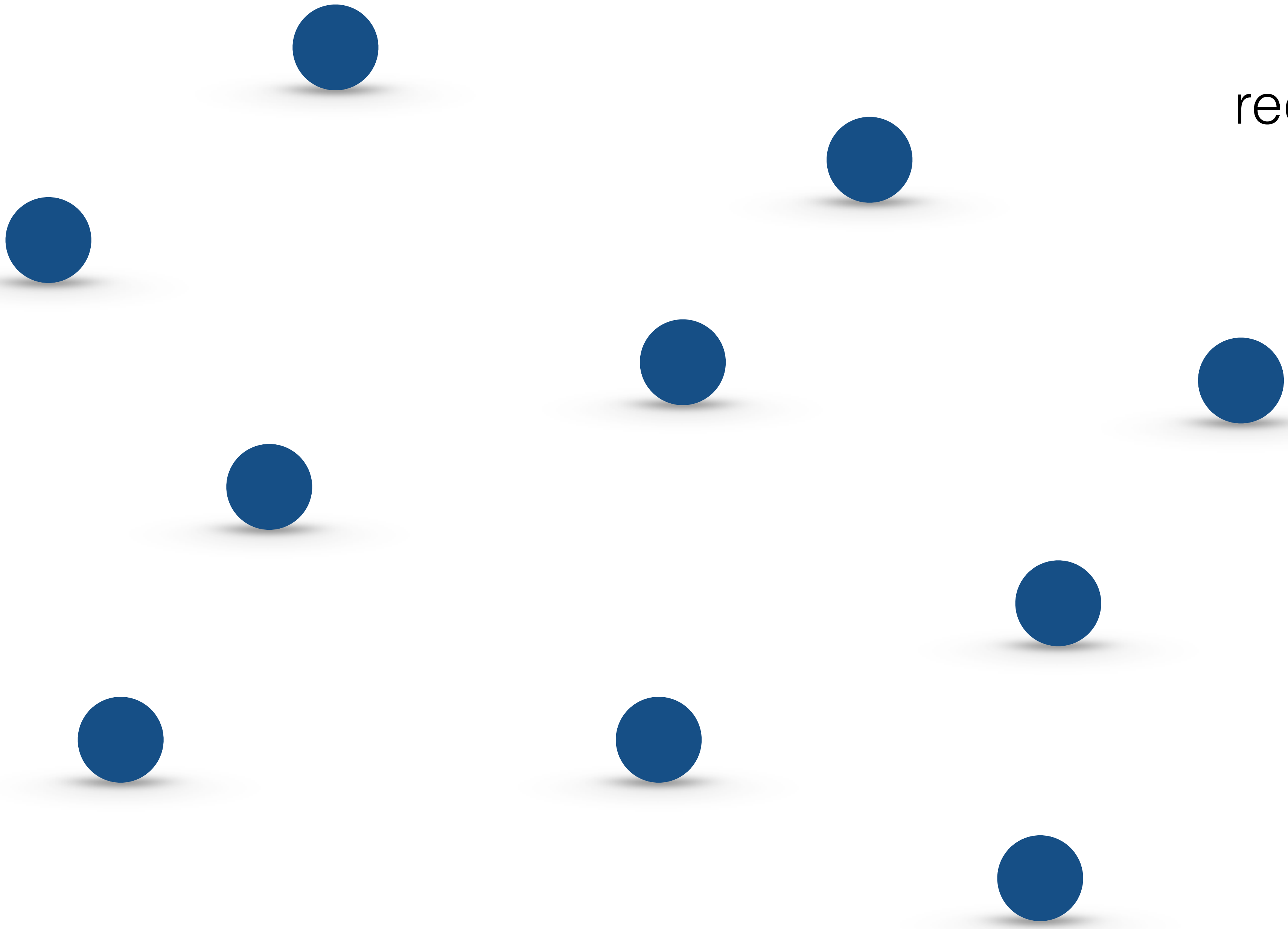

require invariance under:

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$$\vec{z}_i' = T \cdot \begin{pmatrix} \vec{q}_i \\ \vec{p}_i \end{pmatrix}, \quad i = 1, \ldots, N$$

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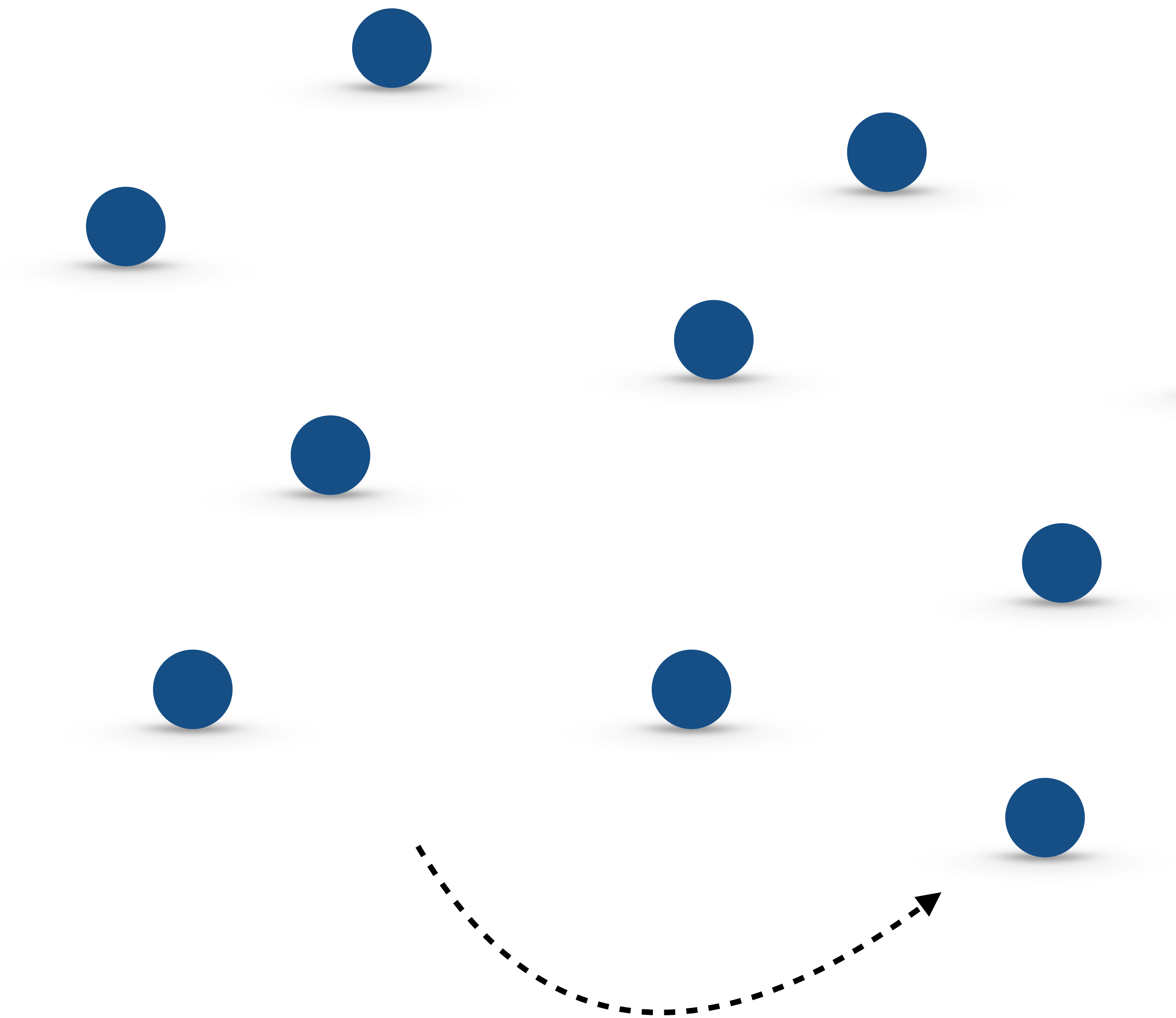

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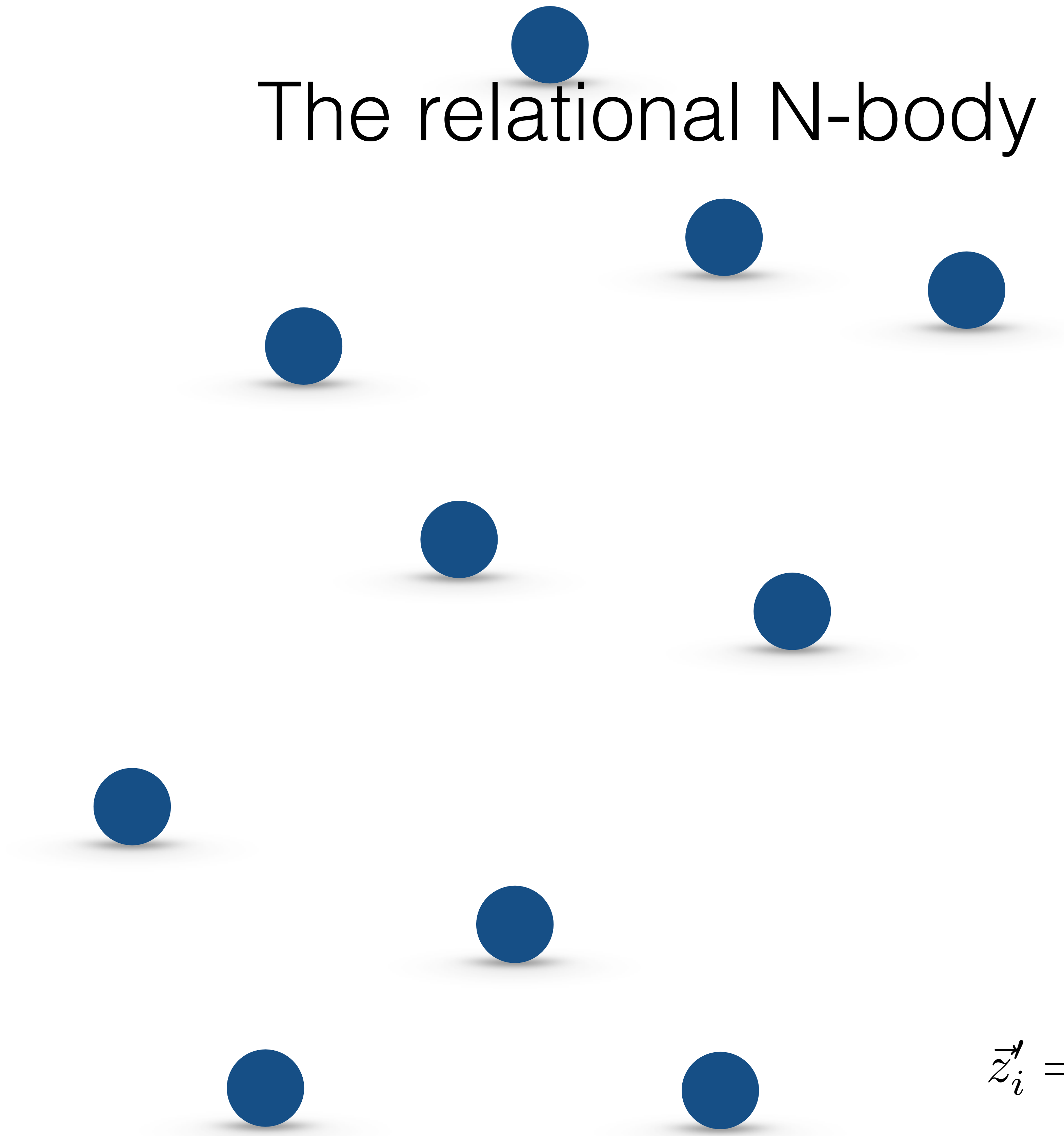

require invariance under:

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$$\vec{z}_i' = R \cdot \begin{pmatrix} \vec{q}_i \\ \vec{p}_i \end{pmatrix}, \quad i = 1, \ldots, N$$

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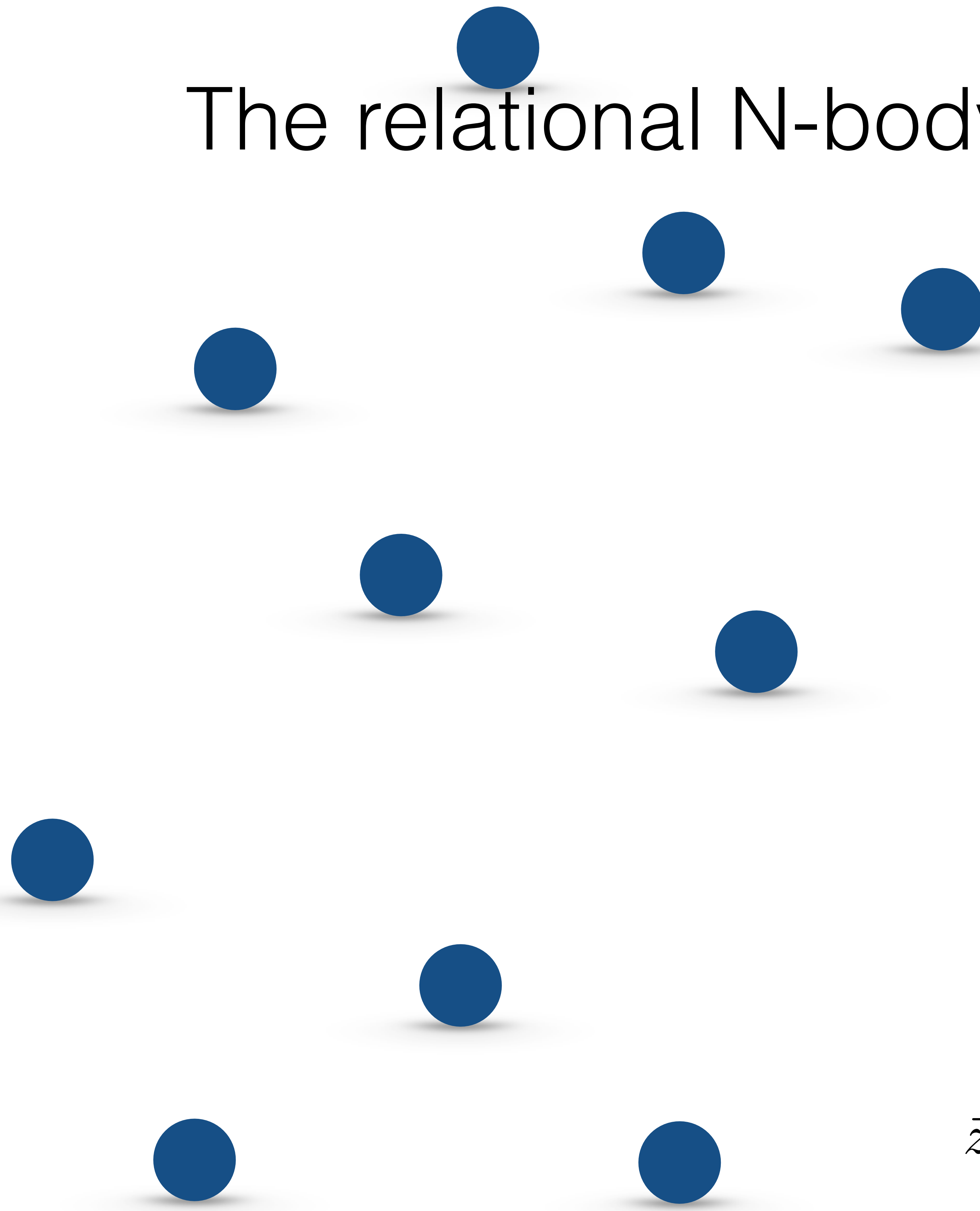

require invariance under:

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$$\Rightarrow \quad \vec{P} = \sum_i \vec{p}_i = 0$$

$$\vec{R} = \sum_i \vec{q}_i \times \vec{p}_i = 0$$

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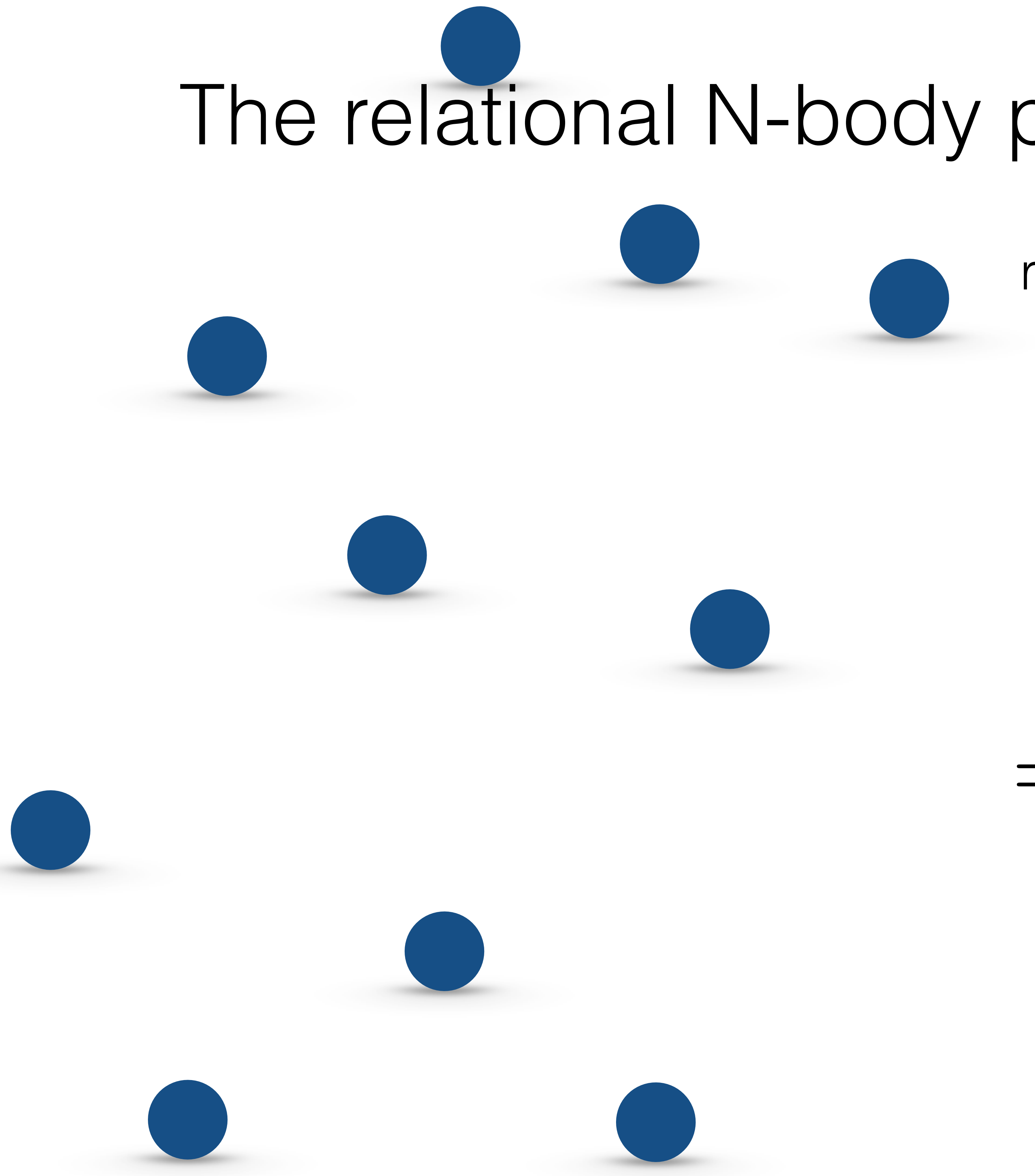

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redundancy in description

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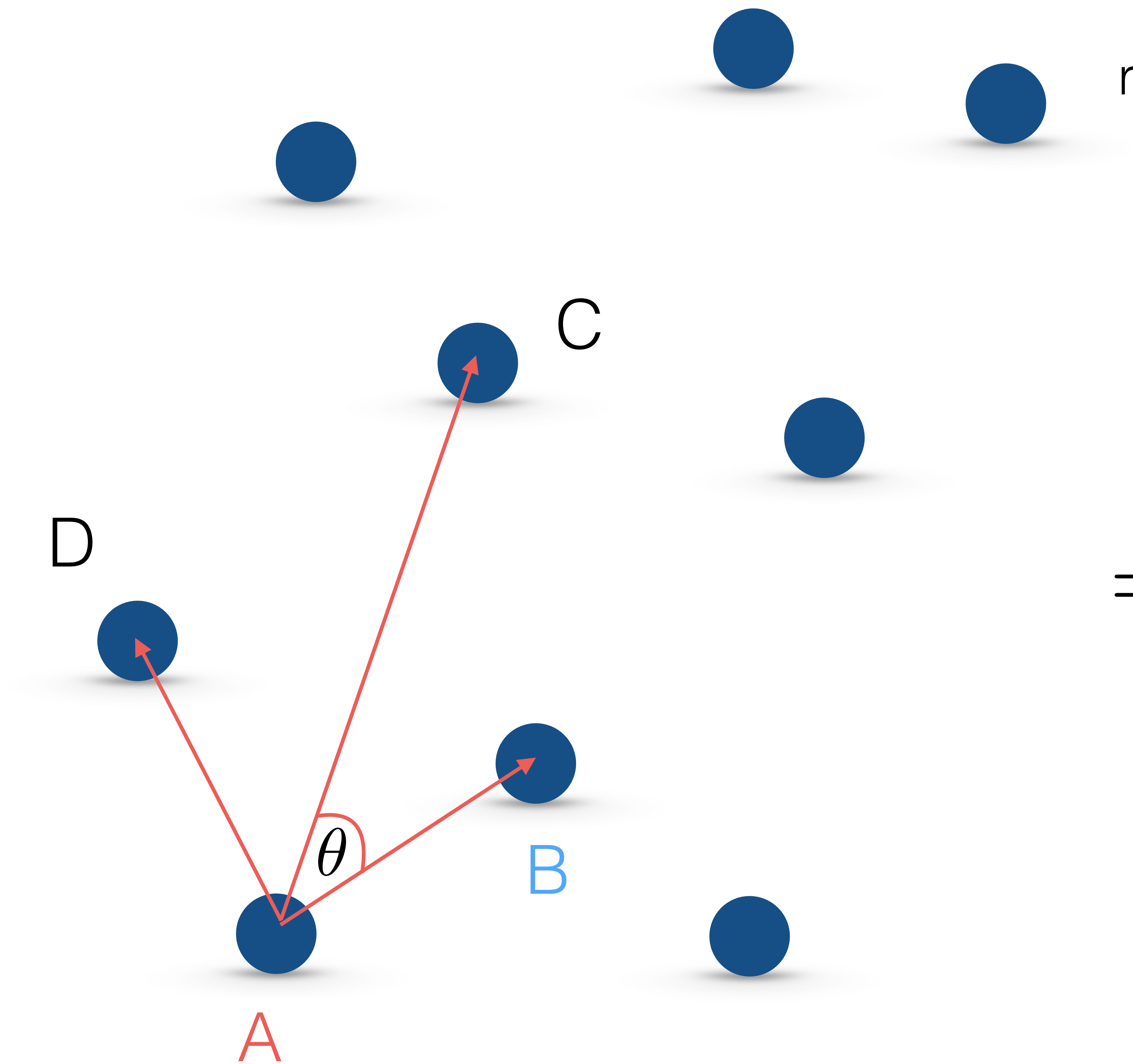

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redundancy in description

physical/gauge-inv. info:
rel. distances & orientation

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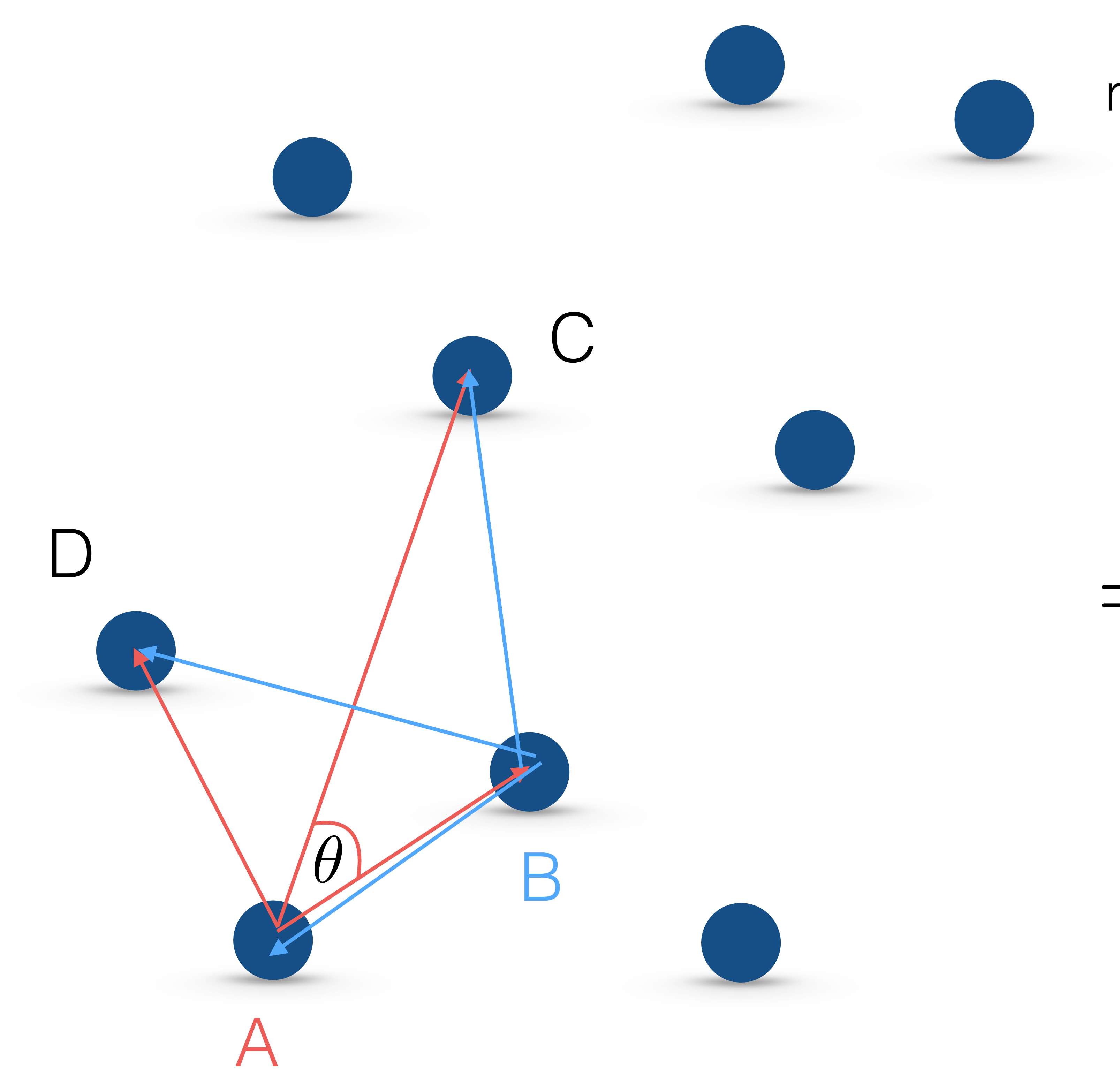

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also of physical info

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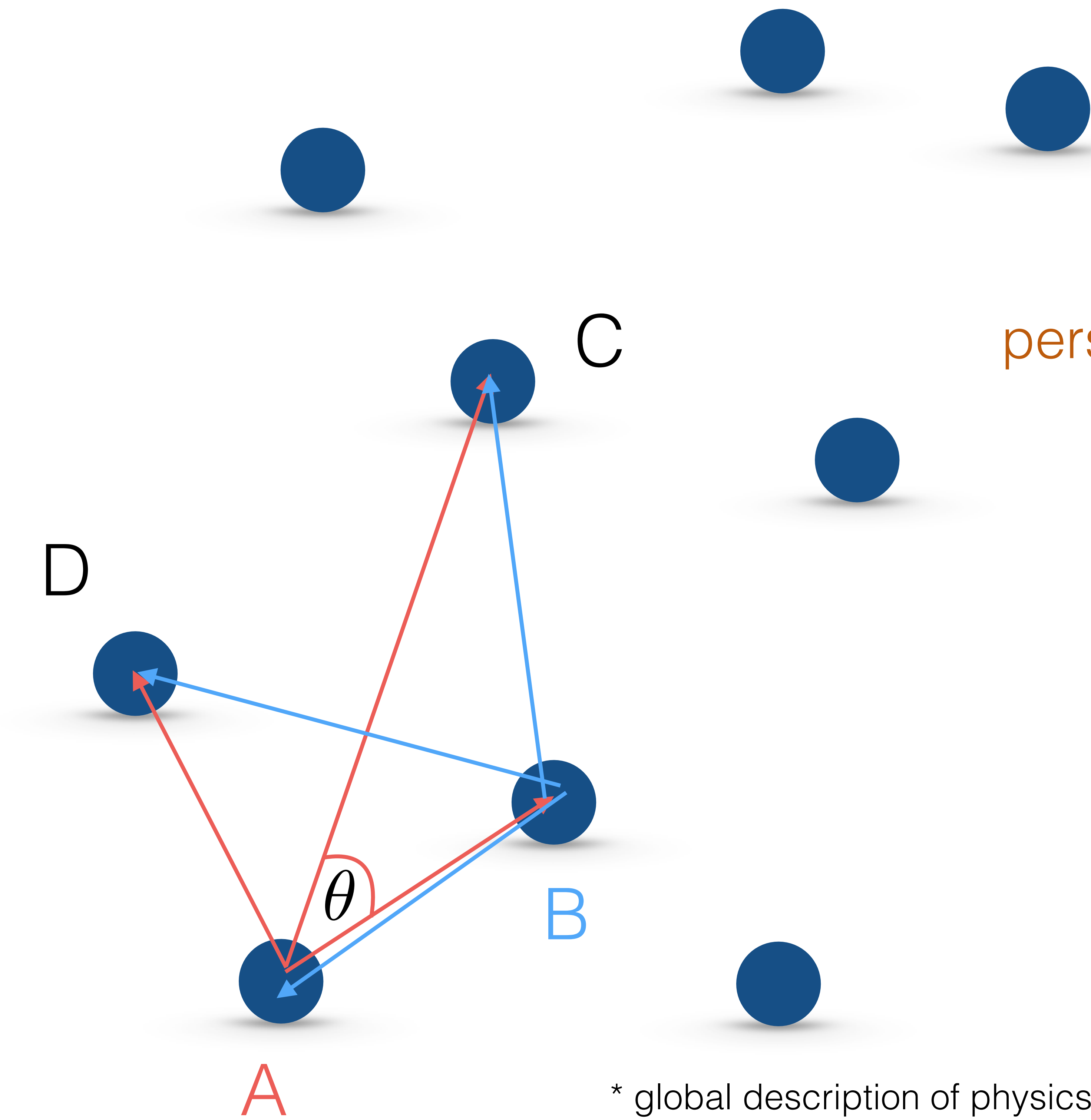

<u>idea:</u>

redundant description

=

perspective-neutral description*

$$\Rightarrow \quad \vec{P} = \sum_i \vec{p}_i = 0$$

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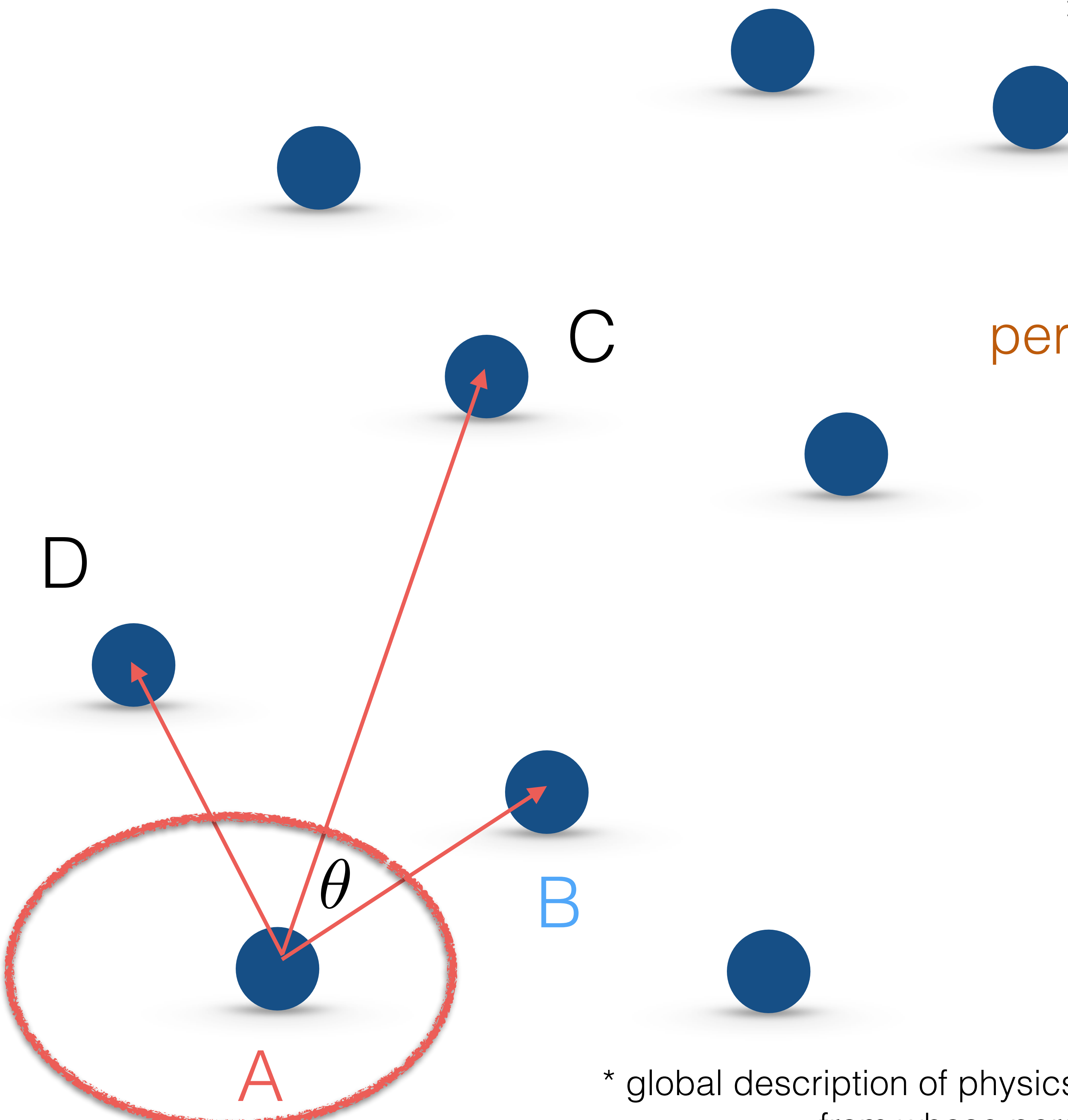
redundancy in description
also of physical info

* global description of physics prior to having chosen a reference frame/system from whose perspective to describe remaining DoFs

Reference system DoFs = redundant DoFs

$\Rightarrow$ avoid self-reference



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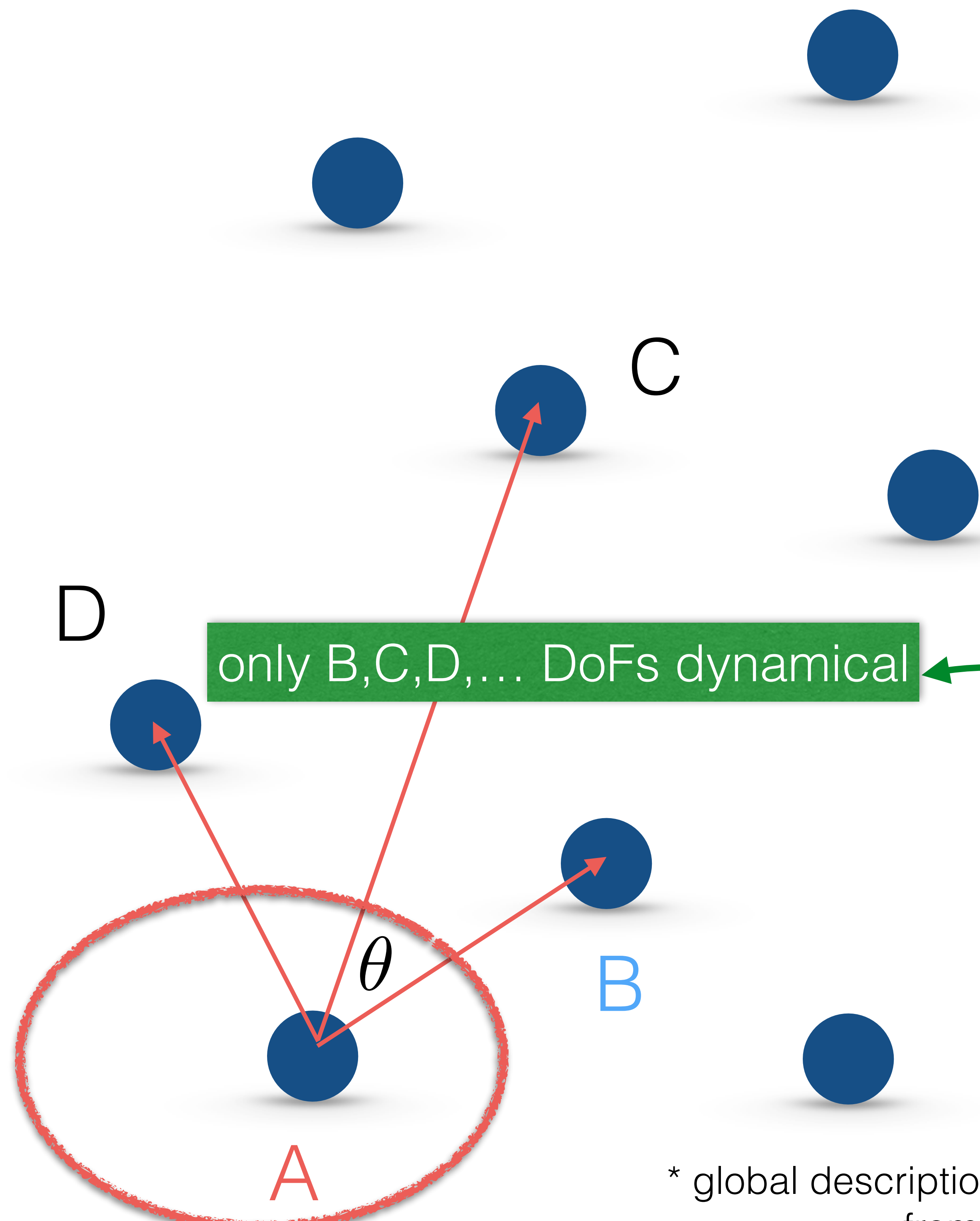

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remove redundancy,
gauge-fix to reference perspective A:

$$\vec{q}_A = 0, \;\; q_B^x = q_B^y = 0 = q_C^y$$

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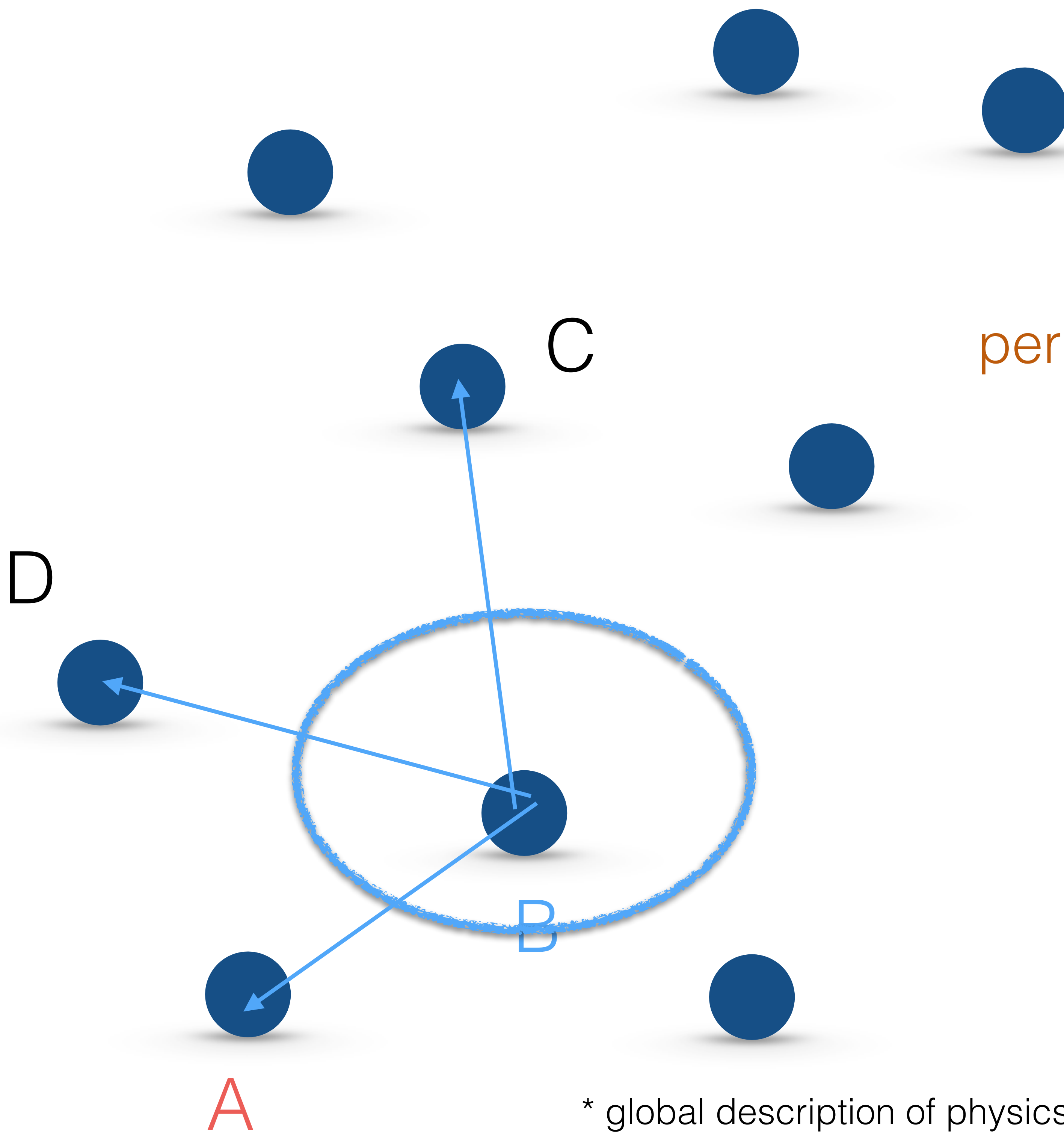
Change of ref. system = gauge transf.



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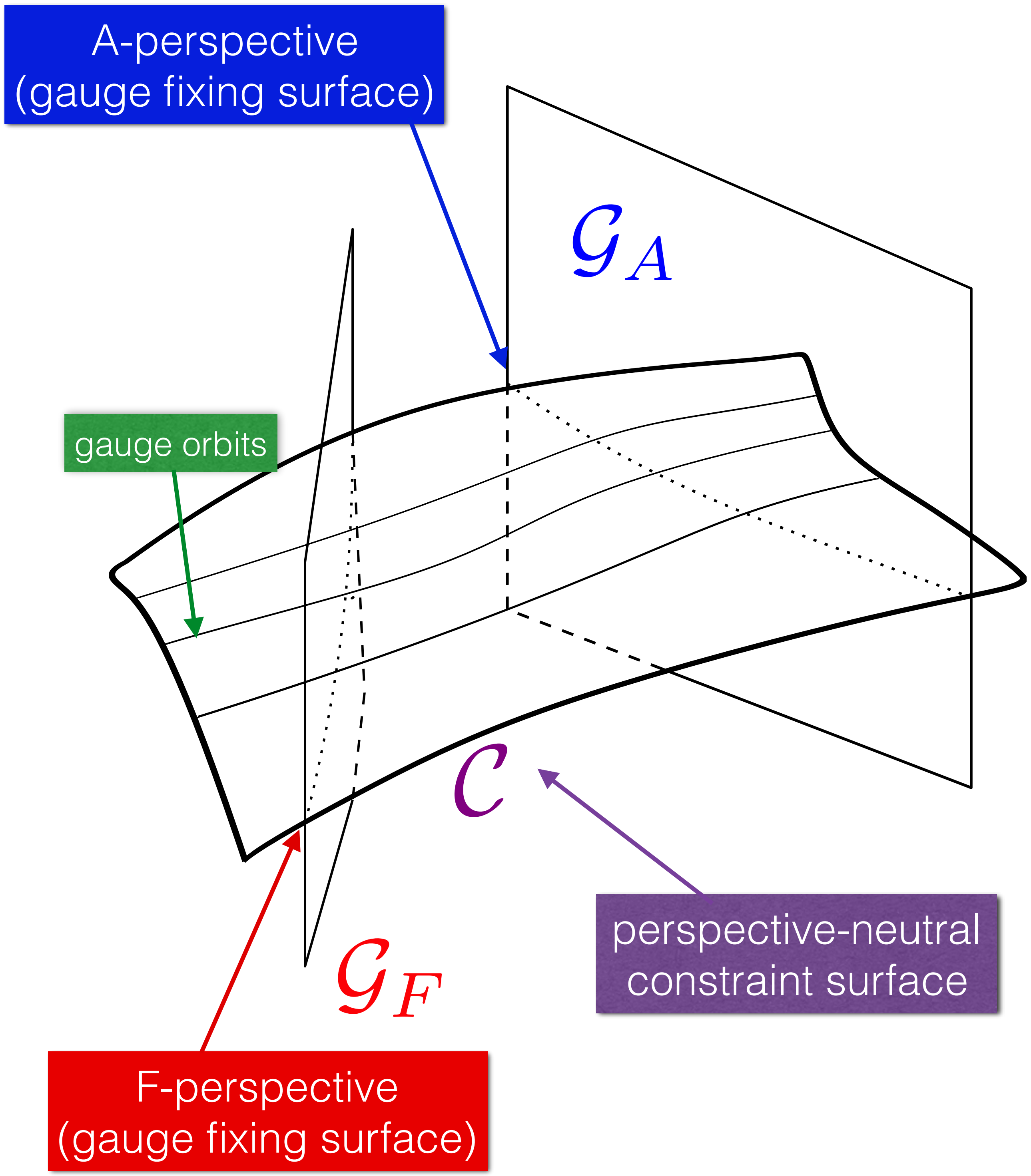
B's perspective, related to A's perspective by gauge transf

* global description of physics prior to having chosen a reference frame/system from whose perspective to describe remaining DoFs

Classical change of frame perspective

Vanrietvelde, PH, Giacomini, Castro-Ruiz, 1809.00556
Vanrietvelde, PH, Giacomini, 1809.05093



perspective-neutral

$$
\begin{tikzcd}
 & \mathcal{C} \arrow[dl, "\text{gauge fix}"'] \arrow[dr, "\text{gauge fix}"] & \\
\mathcal{C} \cap \mathcal{G}_F \arrow[rr, "\text{gauge tr.}"] & & \mathcal{C} \cap \mathcal{G}_A \arrow[d, "\text{proj.}"] \\
\mathcal{P}_F \arrow[u, "\text{embed}"] & & \mathcal{P}_A
\end{tikzcd}
$$

F-perspective (reduced phase space)

A-perspective (reduced phase space)

Perspective-neutral quantum theory

Dirac quantization:

quantize all DoFs on $\mathcal{H}_{\text{kin}} = L^2(\mathbb{R}^{3N})$

solve quantum contraints $\vec{P}\,|\psi\rangle_{\text{phys}} = \vec{R}^2\,|\psi\rangle_{\text{phys}} = 0$

$$\mathcal{H}_{\text{phys}}$$

"quantum constraint surface"

Perspective-neutral quantum theory

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"quantum constraint surface" $\longrightarrow$ $\mathcal{H}_{\rm phys}$

again redundancy in description:

e.g. $(p_A^x + p_B^x + \cdots + p_N^x)\,|\psi\rangle_{\rm phys} = 0$

p_i^x algebraically dependent on $\mathcal{H}_{\rm phys}$ $\Rightarrow$ same for other phys. observables

Perspective-neutral quantum theory

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“quantum constraint surface” $\mathcal{H}_{\text{phys}}$

perspective-neutral QT

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‘Quantum coordinate changes’:

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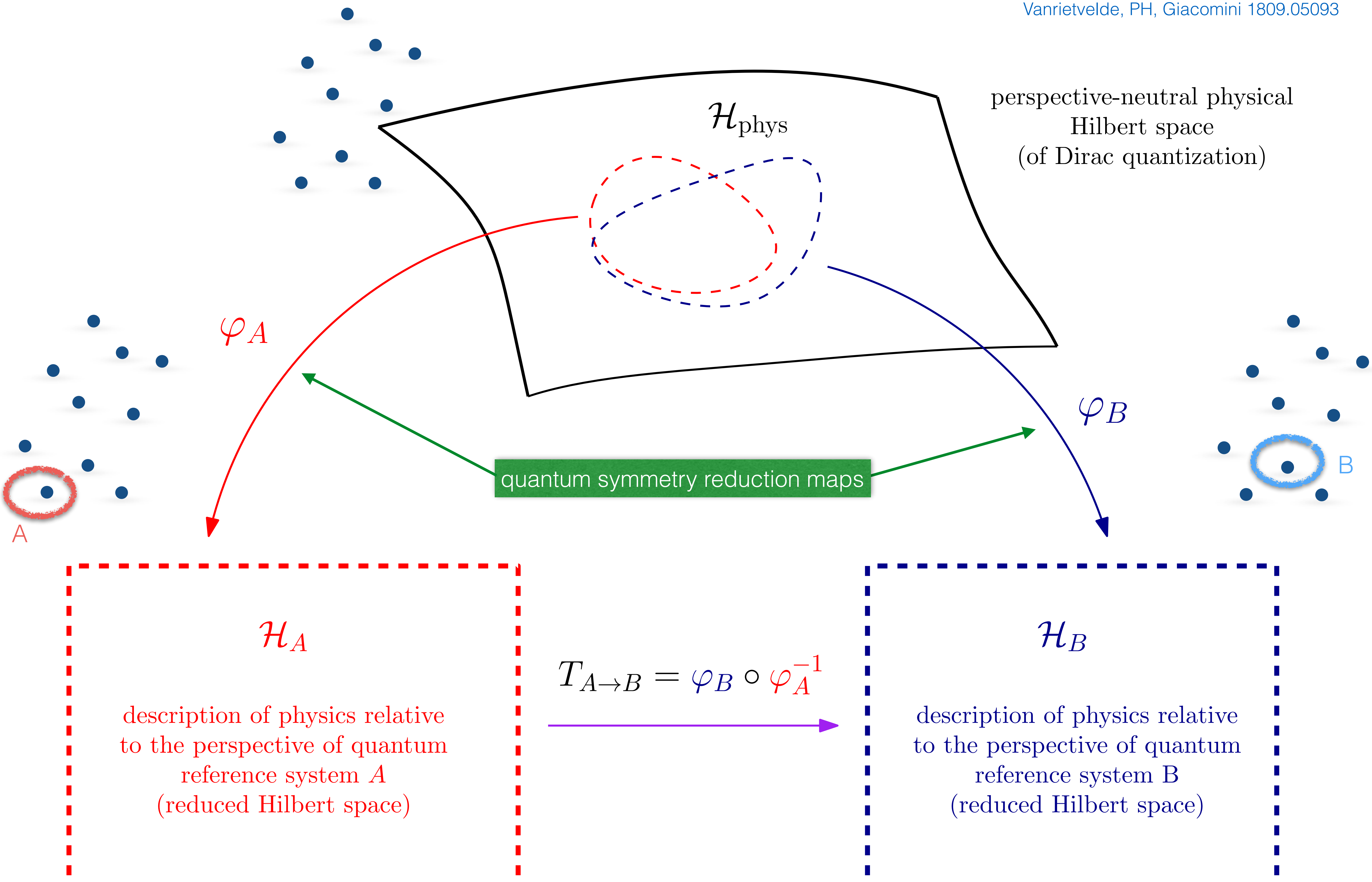

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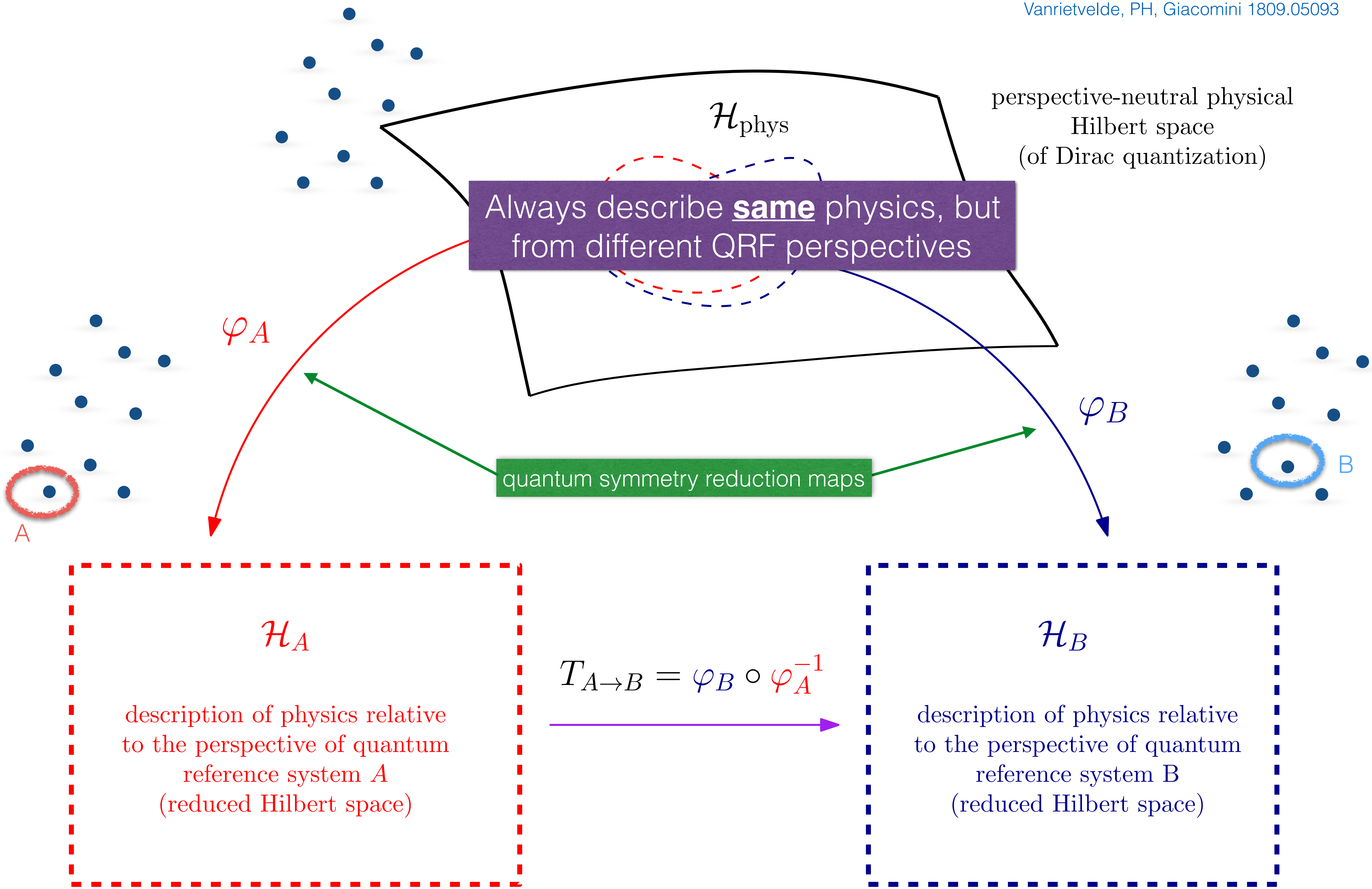
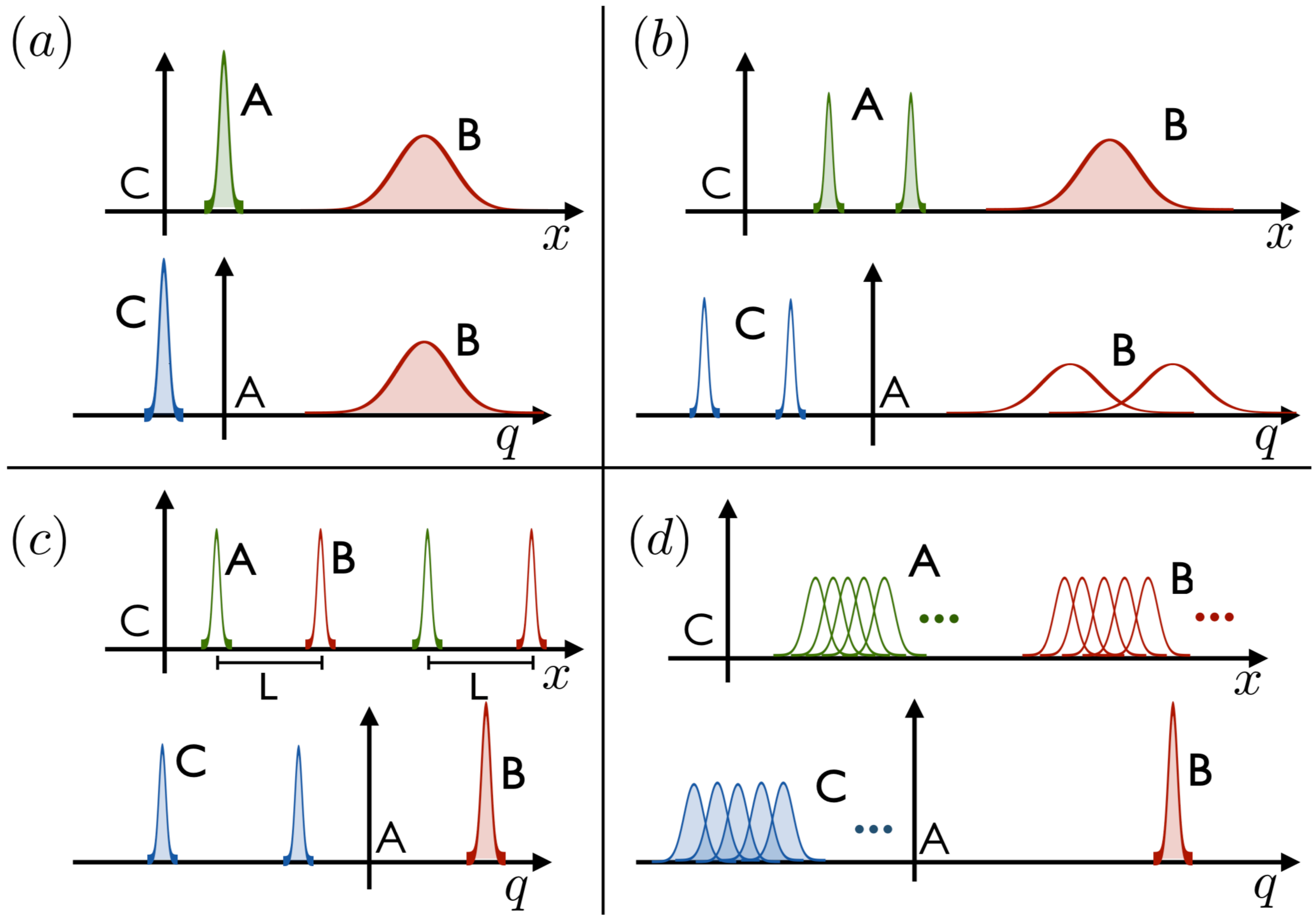


Operational consequences: QRF dependence of correlations and superpositions

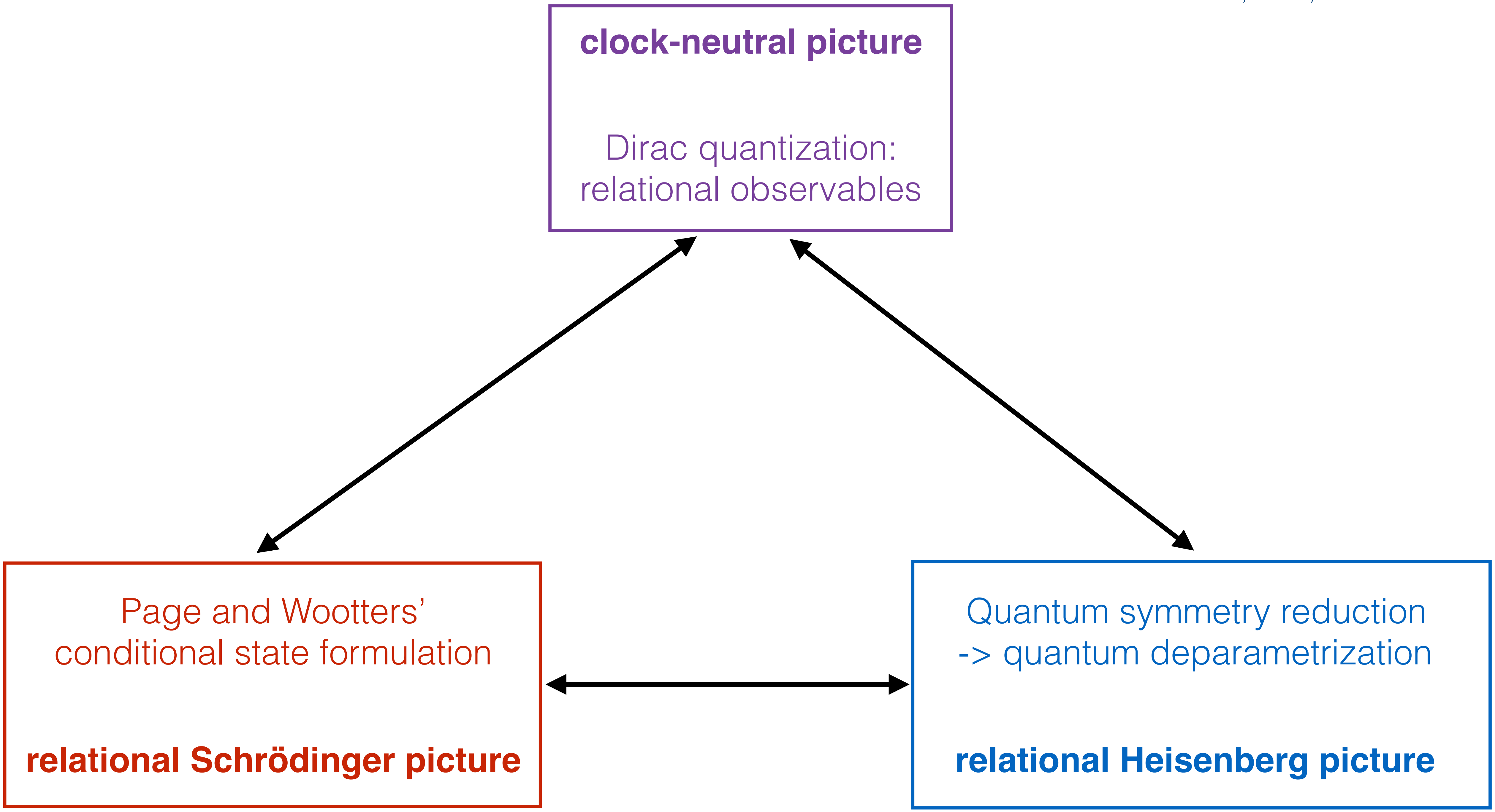


see Giacomini, Castro-Ruiz, Brukner *Nat. Comm.* **10**, 494 (2019)

Application: Quantum Clocks

The trinity of relational quantum dynamics

PH, Smith, Lock 1912.00033



Hamiltonian constraints and time functions

- diffeo & reparametrization inv. lead to Hamiltonian constraints

$$C_H(\{\text{phase space variables}\}) \approx 0$$

$\Rightarrow$ generator of dynamics <u>and</u> symmetry

Example: relativistic particle

$$S_{\text{rel. part.}} = m \int dt \sqrt{-\eta_{\alpha\beta} \frac{dx^\alpha}{dt} \frac{dx^\beta}{dt}}$$

Legendre trans. yields $$C_H = \eta^{\alpha\beta} p_\alpha p_\beta + m^2 \approx 0$$

Hamiltonian constraints and time functions

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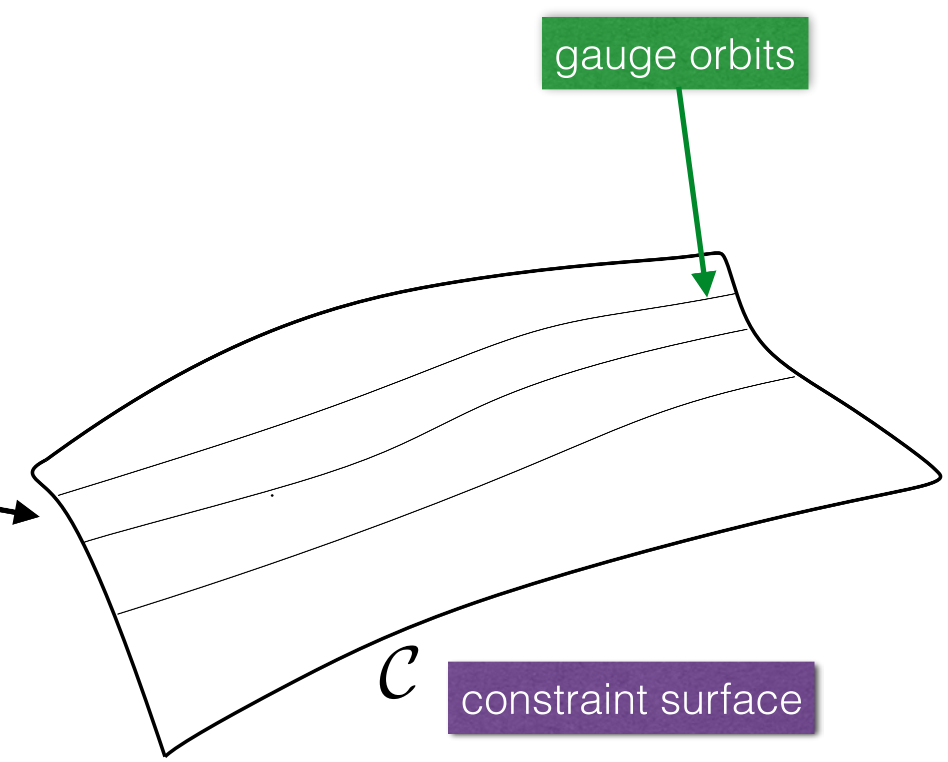
- Redundancy
- Not a phase space



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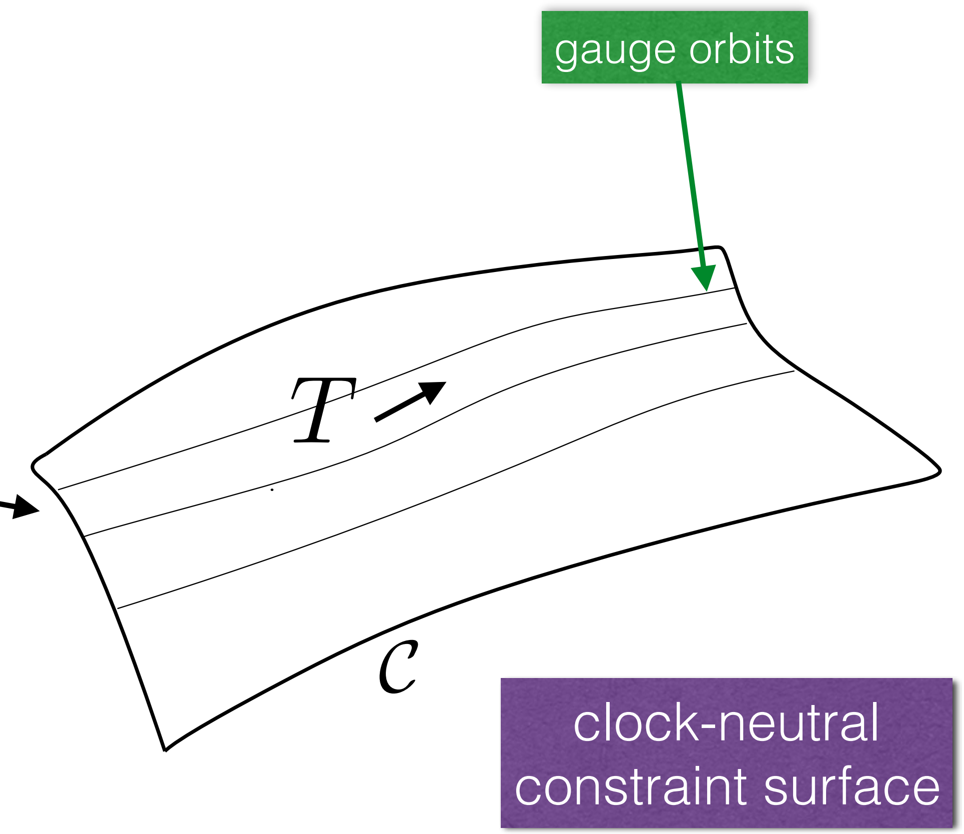


- choose dynamical 'clock' T that parametrizes flow and evolve other DoFs w.r.t. it

[Rovelli; Dittrich,…]

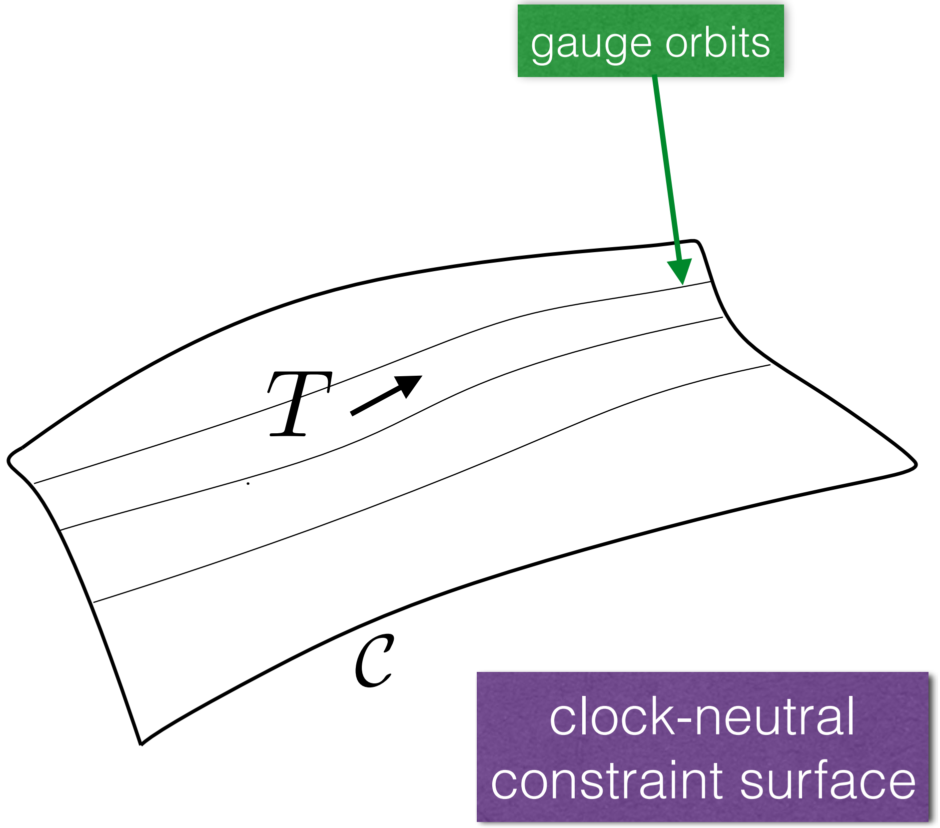
Restrict here to:

- diffeo & reparametrization inv. lead to Hamiltonian constraints

$$C_H(\{\text{phase space variables}\}) \approx 0$$

$\Rightarrow$ generator of dynamics <u>and</u> symmetry

$$C_H = \underbrace{H_C}_{\text{clock}} + \underbrace{H_S}_{\text{system}}$$



- choose dynamical ‘clock’ T that parametrizes flow and evolve other DoFs w.r.t. it

[Rovelli; Dittrich,…]

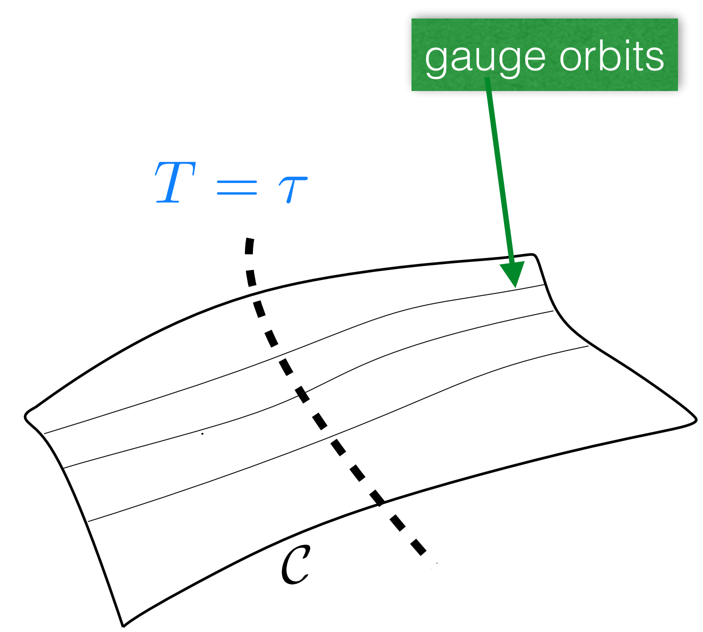
Relational Dirac observables

- under our restrictions:

$$F_{f_S,T}(\tau) = \sum_{n=0} \frac{(T-\tau)^n}{n!} \{H_S, f_S\}_n$$

Rovelli '90s, Dittrich '00s

value of f when $T = \tau$



- gauge-inv. $\{C_H, F_{f_S,T}(\tau)\} \approx 0$

Rovelli: "evolving constants of motion"

gauge-inv. evol. rel. to T
=
"scanning with T=const surfaces through constraint surface"

Problem of time

- Dirac quantization yields

$$\hat{C}_H \left|\psi_{\text{phys}}\right\rangle = (\hat{H}_C + \hat{H}_S) \left|\psi_{\text{phys}}\right\rangle = 0$$

Wheeler-DeWitt type equation, “frozen formalism”

<u>Problem of time:</u>
No more time coordinate, time has to be extracted from quantum DoFs

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$\Longrightarrow$ **relational quantum dynamics**

[DeWitt; Isham; Page & Wootters, Rovelli,…]

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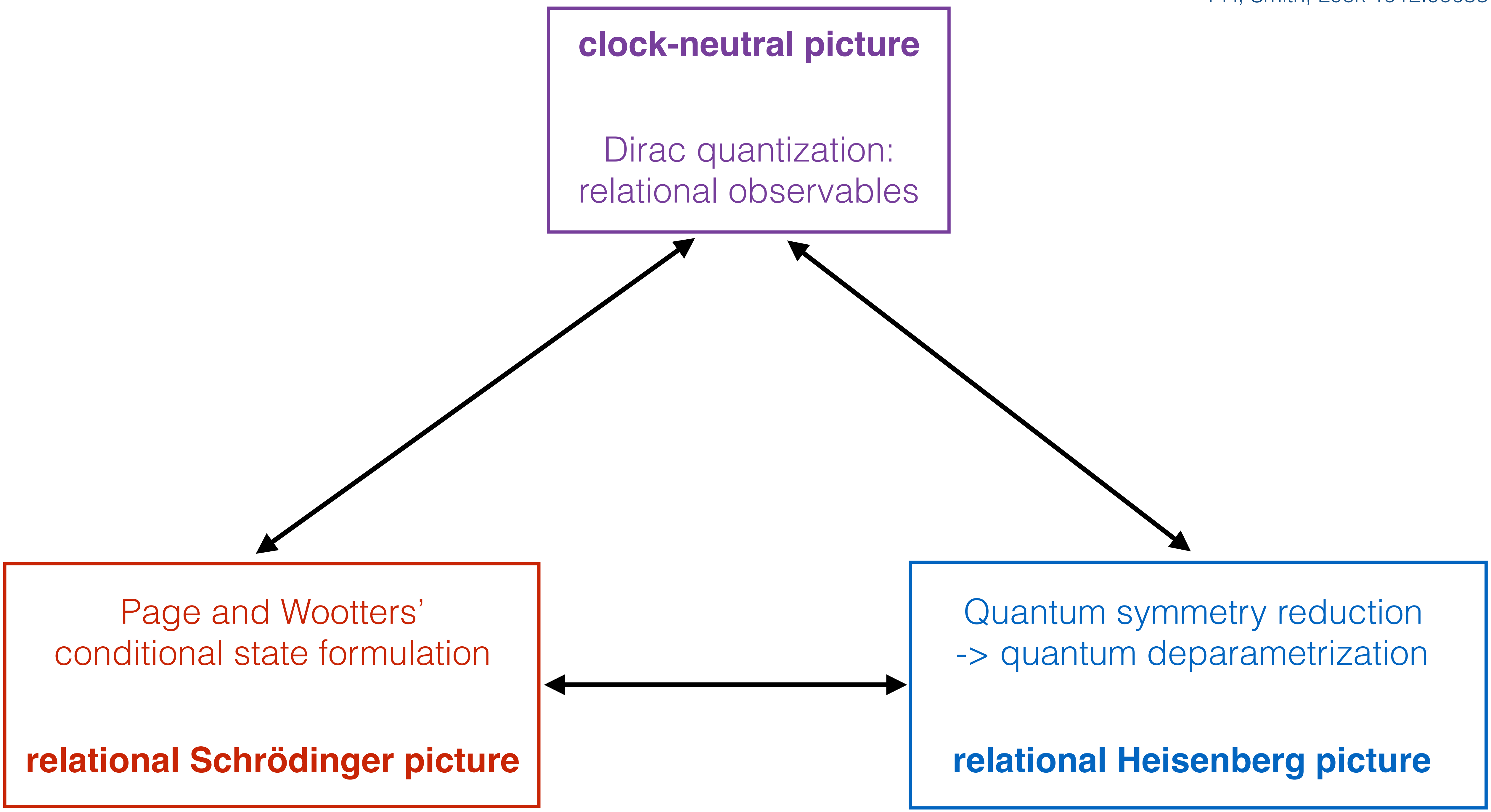
$\Longrightarrow$ **relational quantum dynamics**

[DeWitt; Isham; Page & Wootters, Rovelli,…]

usually: ‘timeless’ state
better: clock-neutral state

The trinity of relational quantum dynamics

PH, Smith, Lock 1912.00033



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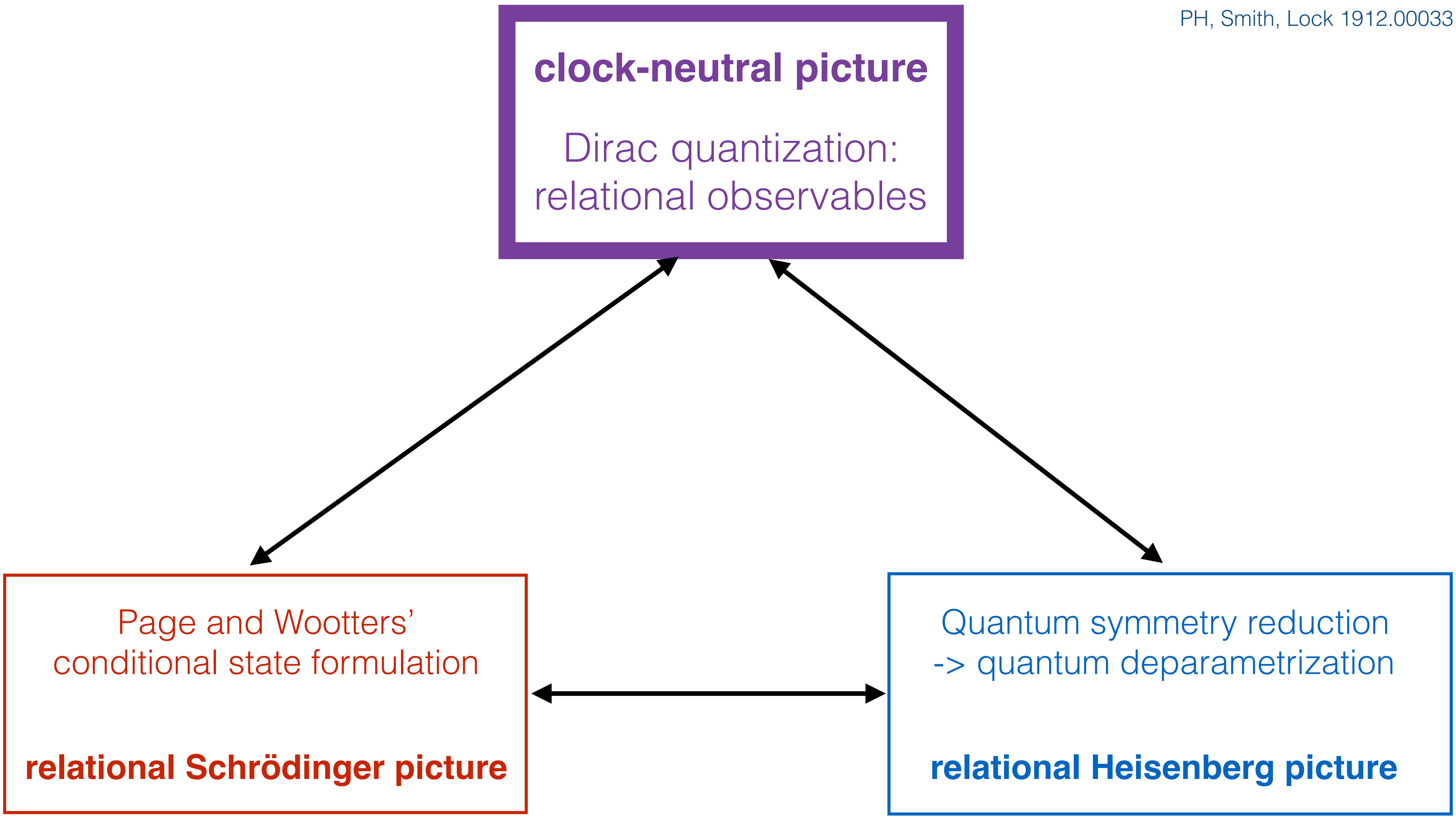

I: Quantum relational Dirac observables

‘clock-neutral’ states

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want to quantize relational observables

$$F_{f_S,T}(\tau) = \sum_{n=0} \frac{(T-\tau)^n}{n!} \{H_S, f_S\}_n$$

$\Rightarrow$ need quantization of T^n

covariant clock POVMs

I: Quantum relational Dirac observables

'clock-neutral' states

$$\hat{C}_H\,|\psi_{\rm phys}\rangle = (\hat{H}_C + \hat{H}_S)\,|\psi_{\rm phys}\rangle = 0$$

- choice of clock, covariant clock states

$$|t\rangle := e^{-i\,t\,\hat{H}_C}\,|t_0\rangle$$ on clock Hilbert space $\mathcal{H}_C$

I: Quantum relational Dirac observables

'clock-neutral' states

$$\hat{C}_H\,|\psi_{\rm phys}\rangle = (\hat{H}_C + \hat{H}_S)\,|\psi_{\rm phys}\rangle = 0$$

- choice of clock, covariant clock states

$$|t\rangle := e^{-i\,t\,\hat{H}_C}\,|t_0\rangle$$ on clock Hilbert space $\mathcal{H}_C$

- dynamics via rel. observables

PH, Smith, Lock 1912.00033

$$\hat{F}_{f,T}(\tau) := \int_{\mathrm{range}(T)} dt\,|t\rangle\langle t|\otimes\sum_{n=0}^{\infty}\frac{i^n}{n!}\,(\tau-t)^n\,\left[\hat{H}_S,\hat{f}_S\right]_n$$

$$= \int_{\mathrm{range}(T)} dt\,e^{-i\,\hat{C}_H\,t}\left(|\tau\rangle\langle\tau|\otimes\hat{f}_S\right)e^{i\,\hat{C}_H\,t}$$ ← 'G-twirl'

gauge-inv.

$$[\hat{F}_{f_S,T},\hat{C}_H] = 0$$

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$$= \int_{\mathrm{range}(T)} dt\, e^{-i\,\hat{C}_H\,t}\left(|\tau\rangle\langle\tau| \otimes \hat{f}_S\right) e^{i\,\hat{C}_H\,t}$$

'G-twirl'

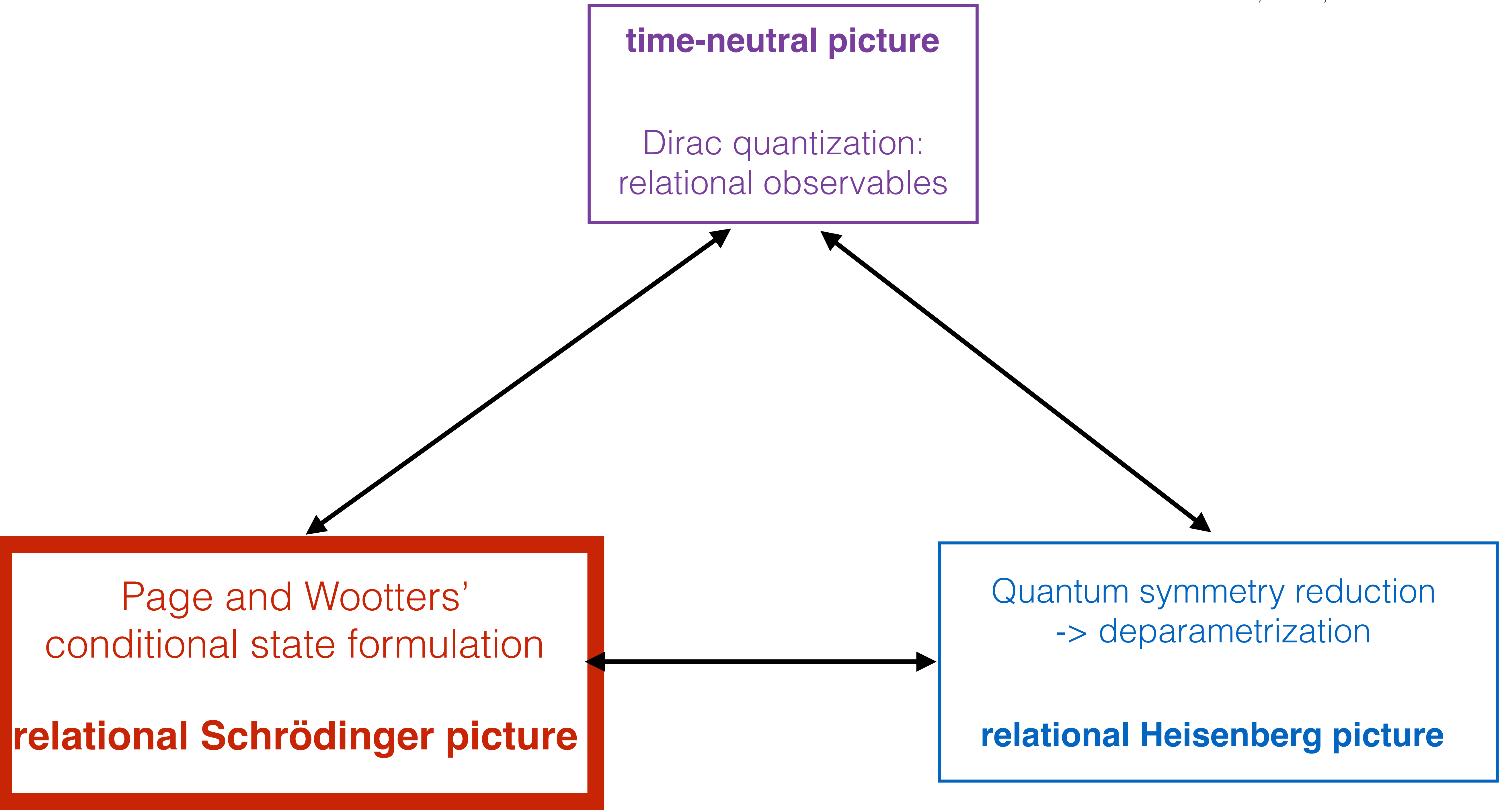
$\Rightarrow$ gauge-inv. evolution: let τ run in

inner product on $\mathcal{H}_{\rm phys}$

$$\langle\psi_{\rm phys}|\hat{F}_{f_S,T}(\tau)|\psi_{\rm phys}\rangle_{\rm phys}$$

The trinity of relational quantum dynamics

PH, Smith, Lock 1912.00033



II: Page-Wootters formalism

Page, Wootters '83

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- conditional state of system when clock reads τ

$$\left|\psi_S(\tau)\right\rangle := \left(\left\langle\tau\right| \otimes I_S\right) \left|\psi_{\text{phys}}\right\rangle$$

- solves **relational Schrödinger eq.**

$$i\,\frac{d}{d\tau} \left|\psi_S(\tau)\right\rangle = \hat{H}_S \left|\psi_S(\tau)\right\rangle$$

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$$i\,\frac{d}{d\tau}\,|\psi_S(\tau)\rangle = \hat{H}_S\,|\psi_S(\tau)\rangle$$

$\Rightarrow$ evaluate system observables $\hat{f}_S$ on evolv. states, e.g.

inner product on $\mathcal{H}_{\rm kin}$

$$\langle\psi_S(\tau)|\,\hat{f}_S\,|\psi_S(\tau)\rangle = \langle\psi_{\rm phys}|(|\tau\rangle\langle\tau| \otimes \hat{f}_S)|\psi_{\rm phys}\rangle_{\rm kin}$$

Kuchar's 3 criticisms of PW formalism

Kuchar 1992

1. violation of constraint

inner product on $\mathcal{H}_{\mathrm{kin}}$

$$\langle\psi_S(\tau)|\,\hat{f}_S\,|\psi_S(\tau)\rangle = \langle\psi_{\mathrm{phys}}|\underline{(|\tau\rangle\langle\tau|\otimes\hat{f}_S)}|\psi_{\mathrm{phys}}\rangle_{\mathrm{kin}}$$

Does not commute with $\hat{C}_H$

$\Rightarrow$ throws $|\psi_{\mathrm{phys}}\rangle$ out of $\mathcal{H}_{\mathrm{phys}}$

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2. wrong propagators

$$P(q' \text{ when } \tau' | q \text{ when } \tau) = \frac{\langle \psi_{\mathrm{phys}}|(|\tau\rangle\langle\tau| \otimes |q\rangle\langle q|)(|\tau'\rangle\langle\tau'| \otimes |q'\rangle\langle q'|)(|\tau\rangle\langle\tau| \otimes |q\rangle\langle q|)|\psi_{\mathrm{phys}}\rangle_{\mathrm{kin}}}{\langle \psi_{\mathrm{phys}}|(|\tau\rangle\langle\tau| \otimes |q\rangle\langle q|)|\psi_{\mathrm{phys}}\rangle_{\mathrm{kin}}}$$

$$= |\delta(\tau - \tau')\delta(q - q')|^2$$

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3. wrong probability distributions for Klein-Gordon systems

(conditioning relative to $\hat{x}_0$)

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$$\langle\psi_S(\tau)|\,\hat{f}_S\,|\psi_S(\tau)\rangle = \langle\psi_{\mathrm{phys}}|(\underline{|\tau\rangle\langle\tau|\otimes\hat{f}_S})|\psi_{\mathrm{phys}}\rangle_{\mathrm{kin}}$$

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effectively ended research on PW formalism for a while

Equivalence of Page-Wootters and Rovelli

PH, Smith, Lock 1912.00033;
+ to appear

conditioning $|\psi_S(\tau)\rangle := (\langle\tau| \otimes I_S)\, |\psi_{\text{phys}}\rangle$ defines reduction

$$\begin{tikzcd} \mathcal{H}_{\text{phys}} \arrow[d, shift right=2, "\mathcal{R}_{\text{PW}}(\tau) = \langle\tau| \otimes I_S"'] \\ \mathcal{H}_S^{\text{phys}} \arrow[u, shift right=2, "\mathcal{R}^{-1}_{\text{PW}}(\tau) = \int_{\mathbb{R}} dt\, |t\rangle \otimes U_S(t-\tau)"'] \end{tikzcd}$$

conditional (Schrödinger) states

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1. equiv. of observables

$$\mathcal{R}_{\text{PW}}^{-1}(\tau)\,\hat{f}_S^{\text{phys}}\,\mathcal{R}_{\text{PW}}(\tau) = \hat{F}_{f_S^{\text{phys}},T}(\tau) \quad \text{on} \quad \mathcal{H}_{\text{phys}}$$

relational observables

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$$\langle\psi_S(\tau)|\phi_S(\tau)\rangle = \langle\psi_{\rm phys}|(|\tau\rangle\langle\tau| \otimes I_S)|\phi_{\rm phys}\rangle_{\rm kin} = \langle\psi_{\rm phys}|\phi_{\rm phys}\rangle_{\rm phys}$$

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PW 'gauge-fixed' version of Rovelli

PH, Smith, Lock 1912.00033;
+ to appear

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PH, Smith, Lock 1912.00033;
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manifestly gauge-invariant

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[different from Gambini et al ’09; Giovanetti et al ’15]

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3. right probability distributions for Klein-Gordon systems

conditioning relative to covariant clock POVM

The trinity of relational quantum dynamics

PH, Smith, Lock 1912.00033

clock-neutral picture

Dirac quantization:
relational observables

Page and Wootters'
conditional state formulation

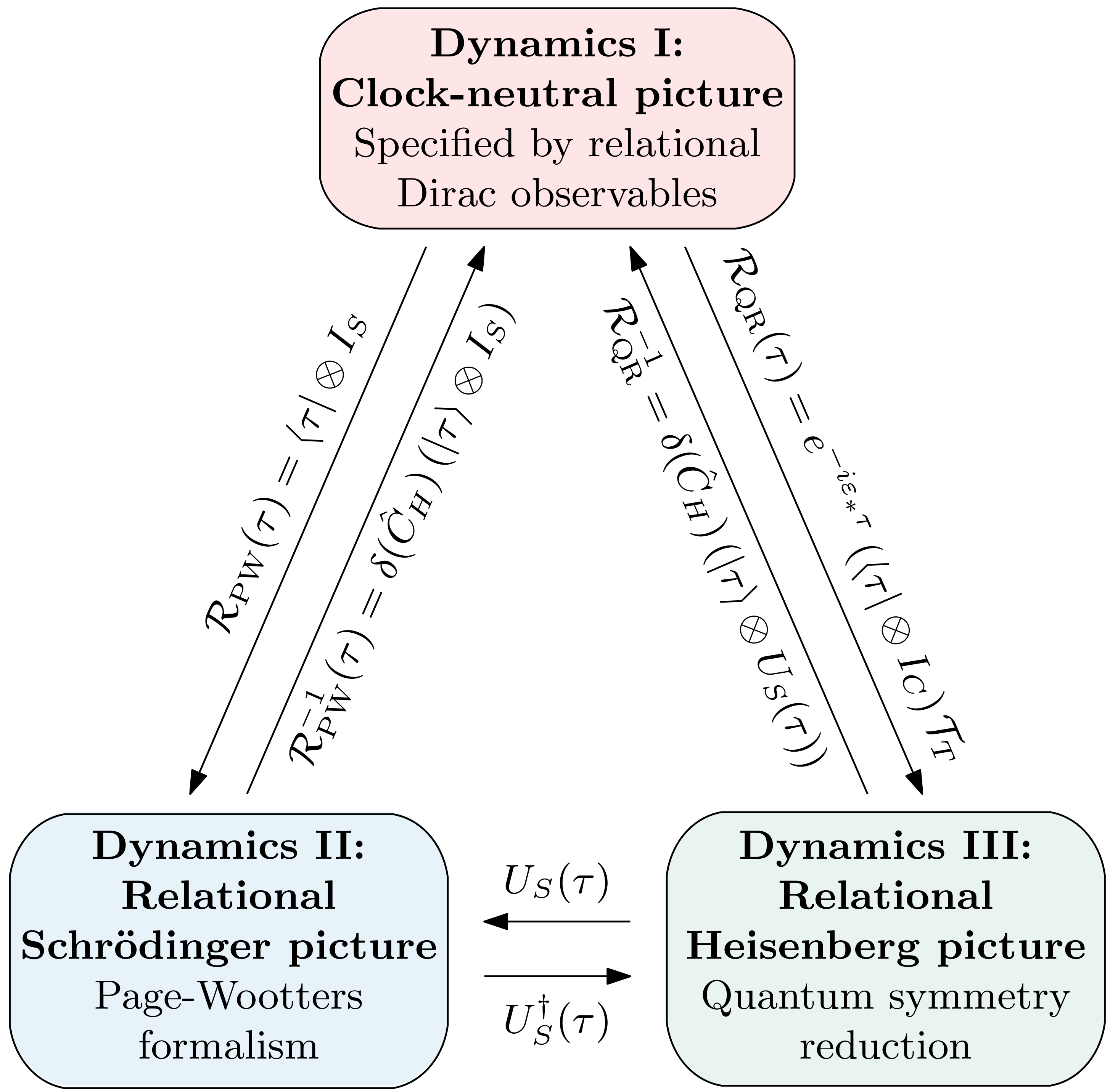
relational Schrödinger picture

Quantum symmetry reduction
-> quantum deparametrization

relational Heisenberg picture

The trinity of relational quantum dynamics

PH, Smith, Lock 1912.00033;
+ to appear



Changing quantum clocks

Bojowald, PH, Tsobanjan, CQG 28, 035006 (2011)
Bojowald, PH, Tsobanjan, PRD 83,125023 (2011)
PH, Kubalova, Tsobanjan, PRD 86, 065014 (2012)

PH, Vanrietvelde 1810.04153
PH 1811.00611
PH, Smith, Lock 1912.00033 + to appear

Multiple choice problem

- many possible choices for relational clocks $\Longrightarrow$ inequivalent quantum dynamics

e.g., 2 clocks variables T_1, T_2

e.g. $T_1 = a, T_2 = \varphi$

in quantum cosmology

$$T_1(T_2) \quad \text{vs} \quad T_2(T_1)$$

What if T_1, T_2 operators?

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"The multiple choice problem is one of an embarrassment of riches: out of many inequivalent options, one does not know which one to select."

Isham (1993):

"Can these different quantum theories be seen to be part of an overall scheme that is covariant?… It seems most unlikely that a single Hilbert space can be used for all possible choices of an internal time function."

Scheme: switching quantum clocks

PH, Vanrietvelde, arxiv:1810.04153
PH, arXiv:1811.00611
PH, Smith, Lock 1912.00033 + to appear

clock-neutral picture

$|\psi_{\mathrm{phys}}\rangle$

Dirac quantization:
relational observables

PW or symmetry reduction

$\varphi_{T_1}(\tau_1)$

PW or symmetry reduction

$\varphi_{T_2}(\tau_2)$

evolution in τ_1

relational Schrödinger/ Heisenberg picture

evolution in τ_2

relational Schrödinger/ Heisenberg picture

$$\hat{C}_H = \underbrace{\hat{H}_1}_{\text{clock 1}} + \underbrace{\hat{H}_2}_{\text{clock 2}} + \hat{H}_S$$

encompasses various classes of non-relativistic and both special and general relativistic models

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PH, Vanrietvelde, arxiv:1810.04153
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$\varphi_{T_1}^{-1}$

PW or symmetry reduction

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$$\hat{C}_H = \hat{H}_1 + \hat{H}_2 + \hat{H}_S$$

clock 1 clock 2

encompasses various classes of non-relativistic and both special and general relativistic models

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PH, Vanrietvelde, arxiv:1810.04153
PH, arXiv:1811.00611
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$$\mathcal{S}_{T_1 \to T_2} = \varphi_{T_2}(\tau_2) \circ \varphi_{T_1}^{-1}$$

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clock 1

clock 2

encompasses various classes of non-relativistic and both special and general relativistic models

Scheme: switching quantum clocks

PH, Vanrietvelde, arxiv:1810.04153
PH, arXiv:1811.00611
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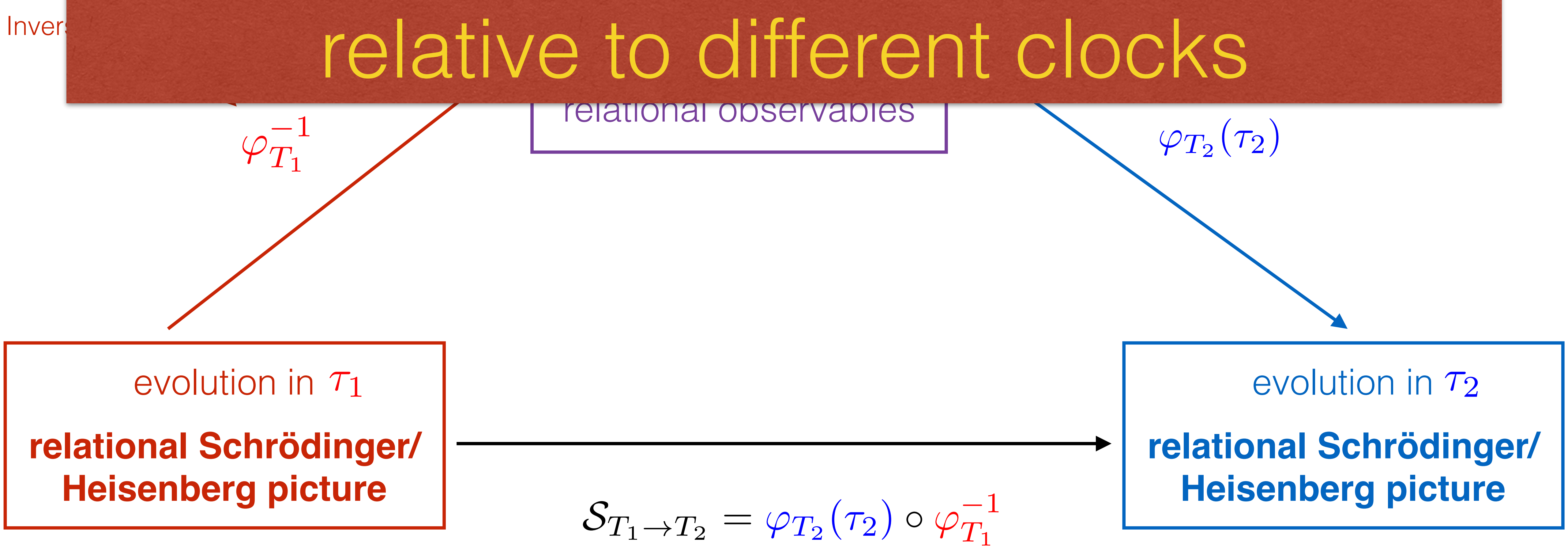
clock 1 clock 2

can show:
observables and states transform correctly

Scheme: switching quantum clocks

PH, Vanrietvelde, arxiv:1810.04153
PH, arXiv:1811.00611
PH, Smith, Lock 1912.00033 + to appear

Always describe **same** dynamics, but relative to different clocks



$$\hat{C}_H = \hat{H}_1 + \hat{H}_2 + \hat{H}_S$$

clock 1 clock 2

can show:
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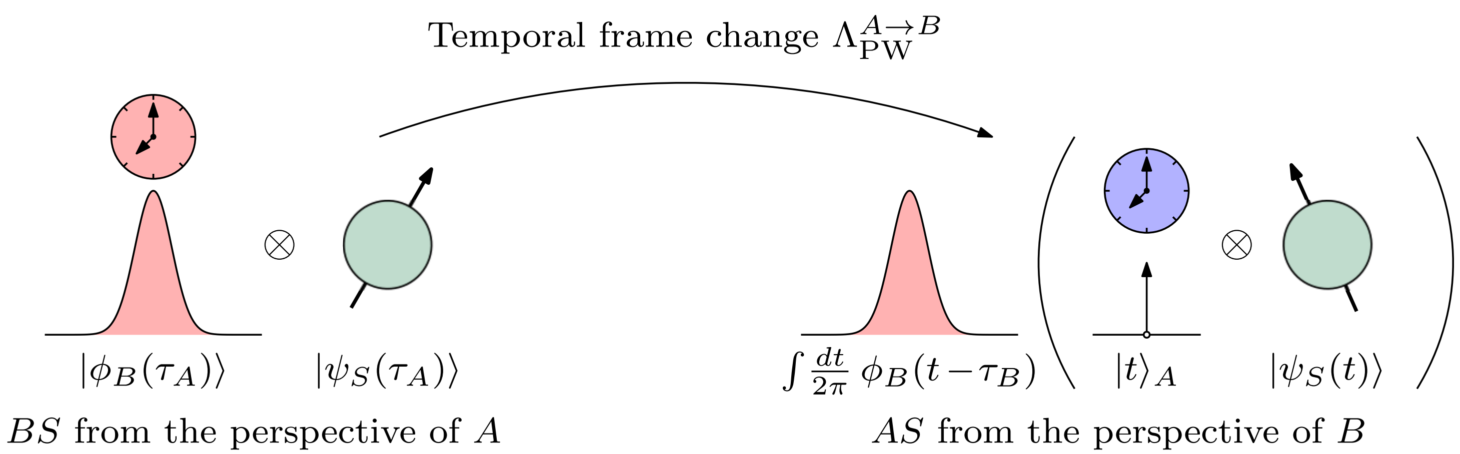
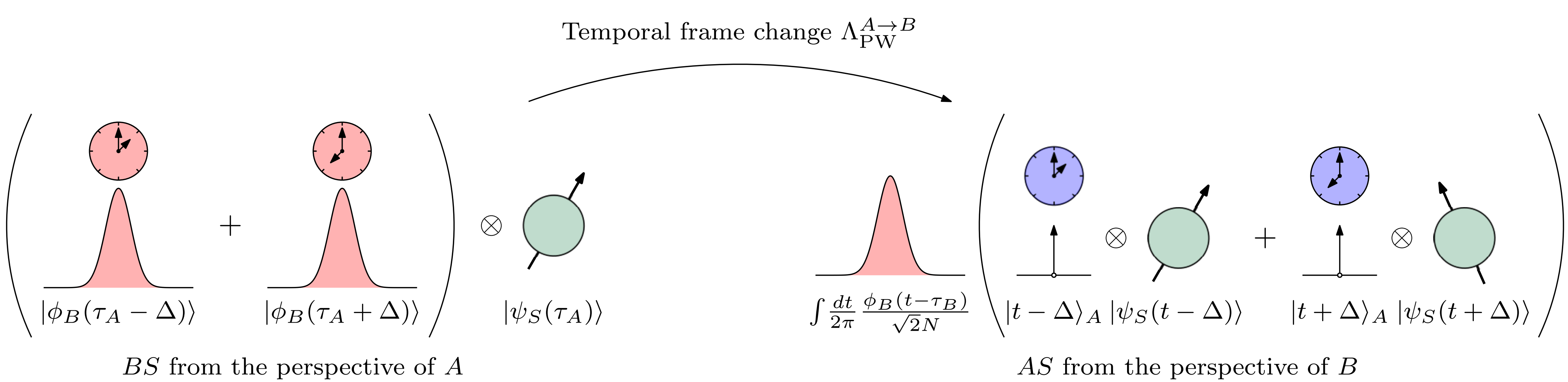
Temporal non-locality

PH, Smith, Lock 1912.00033
Castro-Ruiz et al 1908.10165





superposition of time evolutions

Multiple choice problem

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in quantum cosmology

$T_1(T_2)$ vs $T_2(T_1)$

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Multiple choice ~~problem~~ feature?

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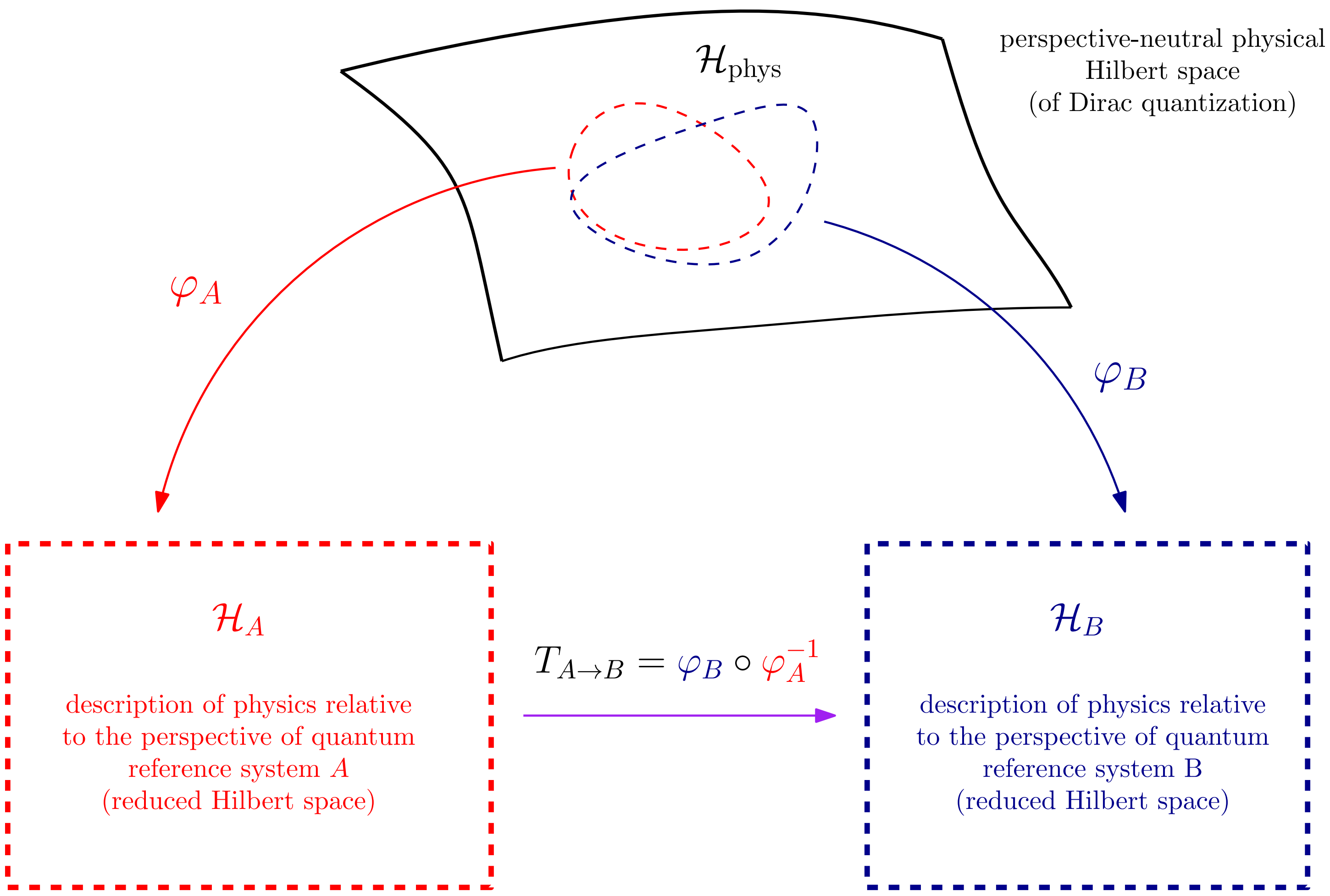
Conclusions



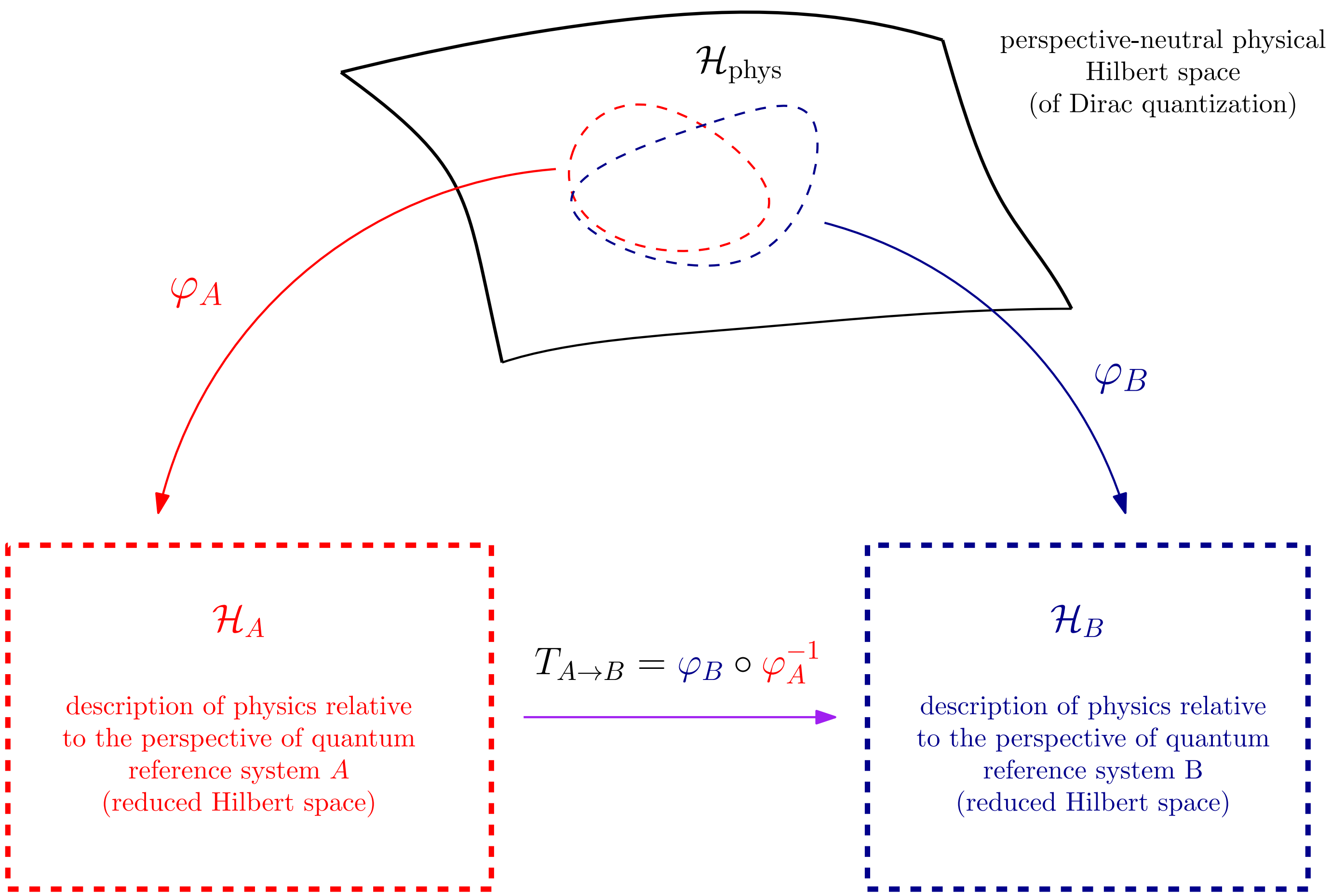
systematic changes of quantum reference system perspectives

- key: quantum reduction method
- both spatial and temporal

$\Rightarrow$

- structure for exploring **quantum version of general covariance**
- **new perspective on problem of time**
  (i) trinity: 3 faces of same rel. quantum dynamics
  (ii) multiple choice feature rather than problem

Outlook



Systematic method for switching
(spatial and temporal)
quantum reference system perspectives

Vanrietvelde, PH, Giacomini, Castro-Ruiz 1809.00556
Vanrietvelde, PH, Giacomini 1809.05093
PH, Vanrietvelde 1810.04153
PH 1811.00611
PH, Smith, Lock 1912.00033 + to appear

applications/extensions:

- QFT
- Measurement problem and Wigner's friend paradox
- Frame dependence of correlations in cosmology
- Import QRF machinery from QI into QG
- New method for constructing quantum rel. Dirac obs.

Appendix

Example: parametrized particle

reparametrization invariant

$$S = \int \mathrm{d}t \, \frac{m}{2} \dot{q}^2 \xrightarrow{\text{extension}} S_{\text{ext}} = \int \mathrm{d}s \, \frac{m}{2} \left(\frac{q'}{t'} \right)^2 t'$$

$\Big\downarrow$ Legendre tr.

$$C_H = p_t + \frac{p^2}{2m} = 0$$

Hamiltonian constraint

generates dynamics <u>and</u> symmetry

Example: parametrized particle

reparametrization invariant

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q	$T(X) = t(s)\,\big\vert_{q(s)=X} = t_0 + \dfrac{X - q_0}{2p}$

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Example: parametrized particle

Getting rid of redundancy through gauge-fixing

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t	$Q(\tau) = q(s)\,\big\vert_{t(s)=\tau} = 2p(\tau - t_0) + q_0$
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Example: parametrized particle

Getting rid of redundancy through gauge-fixing

$$C_H = p_t + \frac{p^2}{2m} = \frac{1}{2m}(p + \sqrt{-p_t})(p - \sqrt{-p_t})$$

clock	relational Dirac observable on constraint surface	reduced observable	gauge fixing:
t	$Q(\tau) = q(s)\,\vert_{t(s)=\tau} = 2p(\tau - t_0) + q_0$	$Q_{\text{red}}(\tau) = 2p\tau + q$	$(t_0 = 0)$
q	$T(X) = t(s)\,\vert_{q(s)=X} = t_0 + \dfrac{X - q_0}{2p}$	$T_{\pm}(X) = t \mp \dfrac{X}{2\sqrt{-p_t}}$	$(q_0 = 0)$

Example: parametrized particle

Getting rid of redundancy through gauge-fixing

$$C_H = p_t + \frac{p^2}{2m} = \frac{1}{2m}(p+\sqrt{-p_t})(p-\sqrt{-p_t})$$

clock	relational Dirac observable on constraint surface	reduced observable on reduced phase space	gauge fixing:
t	$Q(\tau) = q(s)\,\big|_{t(s)=\tau} = 2p(\tau - t_0) + q_0$	$Q_{\mathrm{red}}(\tau) = 2p\tau + q$	$(t_0 = 0)$
q	$T(X) = t(s)\,\big|_{q(s)=X} = t_0 + \dfrac{X - q_0}{2p}$	$T_\pm(X) = t \mp \dfrac{X}{2\sqrt{-p_t}}$	$(q_0 = 0)$

Example: parametrized particle

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$$C_H = p_t + \frac{p^2}{2m} = \frac{1}{2m}(p + \sqrt{-p_t})(p - \sqrt{-p_t})$$

clock	relational Dirac observable on constraint surface	reduced observable on reduced phase space
t	$Q(\tau) = q(s)\,\big\vert_{t(s)=\tau} = 2p(\tau - t_0) + q_0$	evolution w.r.t t $Q_{\mathrm{red}}(\tau) = 2p\tau + q$
q	$T(X) = t(s)\,\big\vert_{q(s)=X} = t_0 + \frac{X - q_0}{2p}$	evolution w.r.t q $T_{\pm}(X) = t \mp \frac{X}{2\sqrt{-p_t}}$

Summary of steps for parametrized particle

PH, Vanrietvelde, arxiv:1810.04153

$$
\begin{tikzcd}
 & & \text{original phase space } \mathbb{R}^4 \arrow[dll, "C_+=q=0"'] \arrow[dl, "C_-=q=0"] \arrow[d, "\text{Dirac quantization}"] \arrow[drr, "C_H=t=0"] & & \\
\mathcal{P}_+ \arrow[d] & \mathcal{P}_- \arrow[d, "\text{affine reduced}"', "\text{quantization}"] & \mathcal{H}_{\text{kin}} \arrow[dd, "\delta(\hat{C}_H)"] & & \mathcal{P}_{q|t} \arrow[dd, "\text{canonical reduced}"', "\text{quantization}"] \\
\mathcal{H}_+ & \mathcal{H}_- & & & \\
 & \mathcal{H}_{\text{phys}}^{t|q} \arrow[ul, "{}_q\langle q=0|\theta(-\hat{p})(-\hat{p}_t)^{\frac{1}{4}}"] \arrow[u, "{}_q\langle q=0|\theta(\hat{p})(-\hat{p}_t)^{\frac{1}{4}}"'] & \mathcal{H}_{\text{phys}} \arrow[l, "\mathcal{T}_q"] \arrow[r, "\mathcal{T}_t"'] & \mathcal{H}_{\text{phys}}^{q|t} \arrow[r, "{}_t\langle t=0|"] & \mathcal{H}_{q|t}
\end{tikzcd}
$$

Quantum clock switch

PH, Vanrietvelde, arxiv:1810.04153

clock-neutral

$$\begin{tikzcd}
 & \mathcal{H}_{\text{phys}} \arrow[dr, "\mathcal{T}_q"] & \\
\mathcal{H}^{q(t)}_{\text{phys}} \arrow[ur, "\mathcal{T}_t^\dagger"] & & \mathcal{H}^{t(q)}_{\text{phys}} \arrow[d, "2\sqrt{\pi}\;{}_q\langle q=0|\theta(\mp\hat{p})(-\hat{p}_t)^{\frac{1}{4}}"] \\
\mathcal{H}_{q(t)} \arrow[u, "|p_t=0\rangle_t"] \arrow[rr, "\hat{\mathcal{S}}_{t\to q\pm}"] & & \mathcal{H}_{\pm t(q)}
\end{tikzcd}$$

evolution w.r.t. t

evolution w.r.t. q

where $\mathcal{T}_q := \mathcal{T}_{q+} + \mathcal{T}_{q-}$ $\qquad$ $\mathcal{T}_{q\pm} := \exp\left(\pm i\,\hat{q}\,(\widehat{\sqrt{-p_t}} - \epsilon)\right)\,\theta(\mp\hat{p})$

$$\mathcal{T}_t := \exp(i\,\hat{t}\,\hat{p}^2/2m) = \exp(i\,\hat{t}\,\hat{H})$$

$$\hat{\mathcal{S}}_{t\to q\pm} \equiv \sqrt{2}\,\hat{\mathcal{P}}_{q\to t}\,\theta(\mp\hat{p})\,\widehat{\sqrt{|p|}}$$

clock swap

Quantum clock switch

PH, Vanrietvelde, arxiv:1810.04153

clock-neutral

$$
\begin{tikzcd}
& \mathcal{H}_{\rm phys} \arrow[dr, "\mathcal{T}_q"] & \\
\mathcal{H}^{q(t)}_{\rm phys} \arrow[ur, "\mathcal{T}_t^\dagger"] & & \mathcal{H}^{t(q)}_{\rm phys} \arrow[d, "2\sqrt{\pi}\;{}_q\langle q{=}0|\theta(\mp\hat{p})(-\hat{p}_t)^{\frac{1}{4}}"] \\
\mathcal{H}_{q(t)} \arrow[u, "|p_t{=}0\rangle_t"] \arrow[rr, "\hat{\mathcal{S}}_{t\to q\pm}"] & & \mathcal{H}_{\pm t(q)}
\end{tikzcd}
$$

evolution w.r.t. t

evolution w.r.t. q

observables transform correctly

$$\mathcal{T}_t \hat{Q}(\tau) \mathcal{T}_t^\dagger = \hat{Q}_{\rm red}(\tau)$$

$$(\widehat{-p_t})^{1/4}\, \mathcal{T}_q\, \hat{T}_\delta(X)\, \mathcal{T}_q^{-1}\, (\widehat{-p_t})^{-1/4} = \hat{T}^\delta_+(X)\,\theta(-\hat{p}) + \hat{T}^\delta_-(X)\,\theta(\hat{p})$$

Regularization details: time-of-arrival

Dirac: affine reduced:

had: $\widehat{(-p_t)^{1/4}}\,\mathcal{T}_q\,\hat{T}_\delta(X)\,\mathcal{T}_q^{-1}\,\widehat{(-p_t)^{-1/4}} = \hat{T}^\delta_+(X)\,\theta(-\hat{p}) + \hat{T}^\delta_-(X)\,\theta(\hat{p})$

Dirac quantization

$$\hat{T}_\delta(X) := \hat{t}_\delta + \frac{1}{4}\left(\widehat{(p)_\delta^{-1}}\,(X-\hat{q}) + (X-\hat{q})\,\widehat{(p)_\delta^{-1}}\right)$$

$$\hat{t}_\delta := \frac{1}{2}\left(\hat{t}\,\hat{p}_t\,\widehat{(p_t)_\delta^{-1}} + \hat{p}_t\,\widehat{(p_t)_\delta^{-1}}\,\hat{t}\right)$$

$$\widehat{(p_t)_\delta^{-1}}|p_t\rangle := \begin{cases} \frac{1}{p_t}|p_t\rangle & p_t \leq -\delta^2, \\ -\frac{1}{\delta^2}|p_t\rangle & -\delta^2 < p_t \leq 0\,, \end{cases}$$

Affine Reduced Quantization

$$\hat{T}^\delta_\pm(X) := \hat{t}_{\delta\pm} \mp \frac{X}{2}\,\widehat{(\sqrt{-p_t})_\delta^{-1}}$$

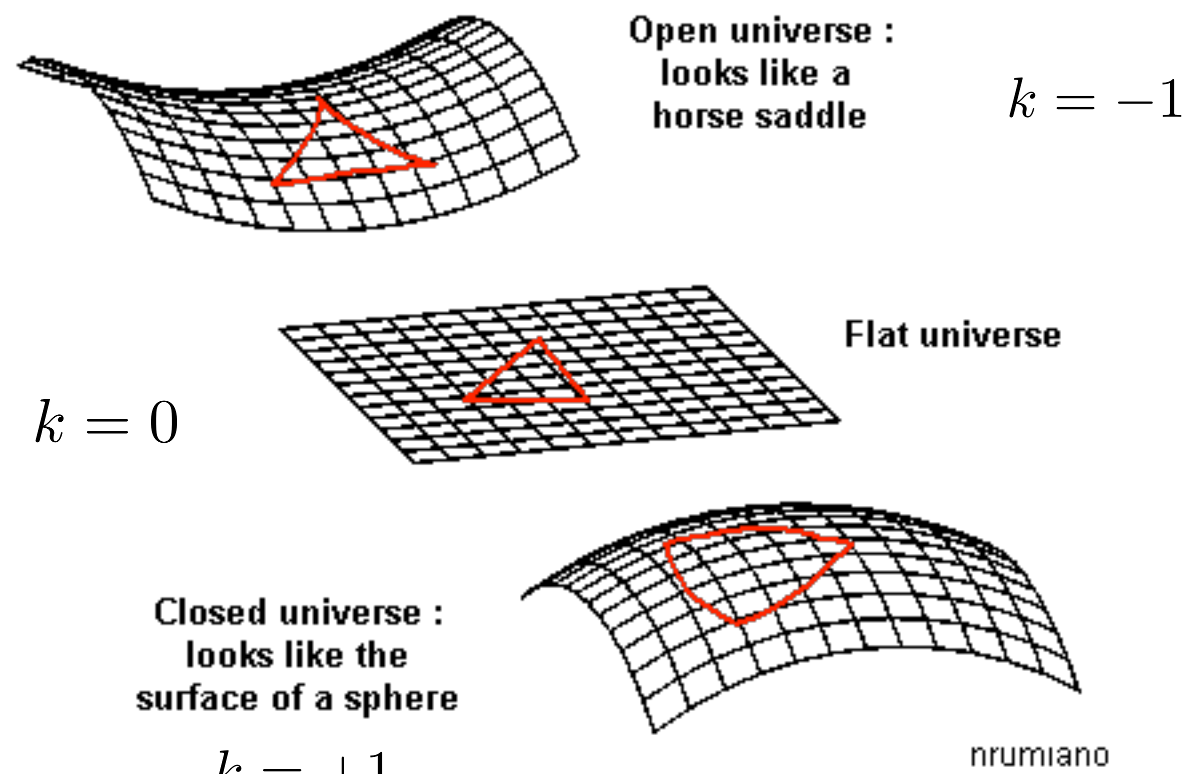
$$\hat{t}_{\delta\pm} := \frac{1}{2}\left(\widehat{(p_t)_\delta^{-1}}\,\hat{\mathfrak{t}} + \hat{\mathfrak{t}}\,\widehat{(p_t)_\delta^{-1}}\right)$$

$$\widehat{(\sqrt{-p_t})_\delta^{-1}}|p_t\rangle_\pm := \begin{cases} \frac{1}{\sqrt{-p_t}}|p_t\rangle_\pm & p_t \leq -\delta^2, \\ \frac{\sqrt{-p_t}}{\delta^2}|p_t\rangle_\pm & -\delta^2 < p_t \leq 0. \end{cases}$$

Relational dynamics in FRW

- homogeneous & isotropic universe

$$ds^2 = -dt^2 + a(t)\left(\frac{dr^2}{1-kr^2} + r^2\,d\Omega^2\right)$$



http://nrumiano.free.fr/Ecosmo/cg_model.html

- Hamiltonian constraint incl. homog. field

$$C_H = p_\phi^2 - p_\alpha^2 - 4k\,e^{4\alpha} + 4m^2\phi^2\,e^{6\alpha} \approx 0$$

$$\alpha = \ln a$$

$\Rightarrow$ time evol. generated is gauge transf.

$$\dot\phi = \{\phi, C_H\} = 2p_\phi$$
$$\dot\alpha = \{\alpha, C_H\} = -2p_\alpha$$

can go through 0, thus not necessarily monotonic

$\Rightarrow$ want to use α or ϕ as relational “clock”

Relational dynamics in FRW

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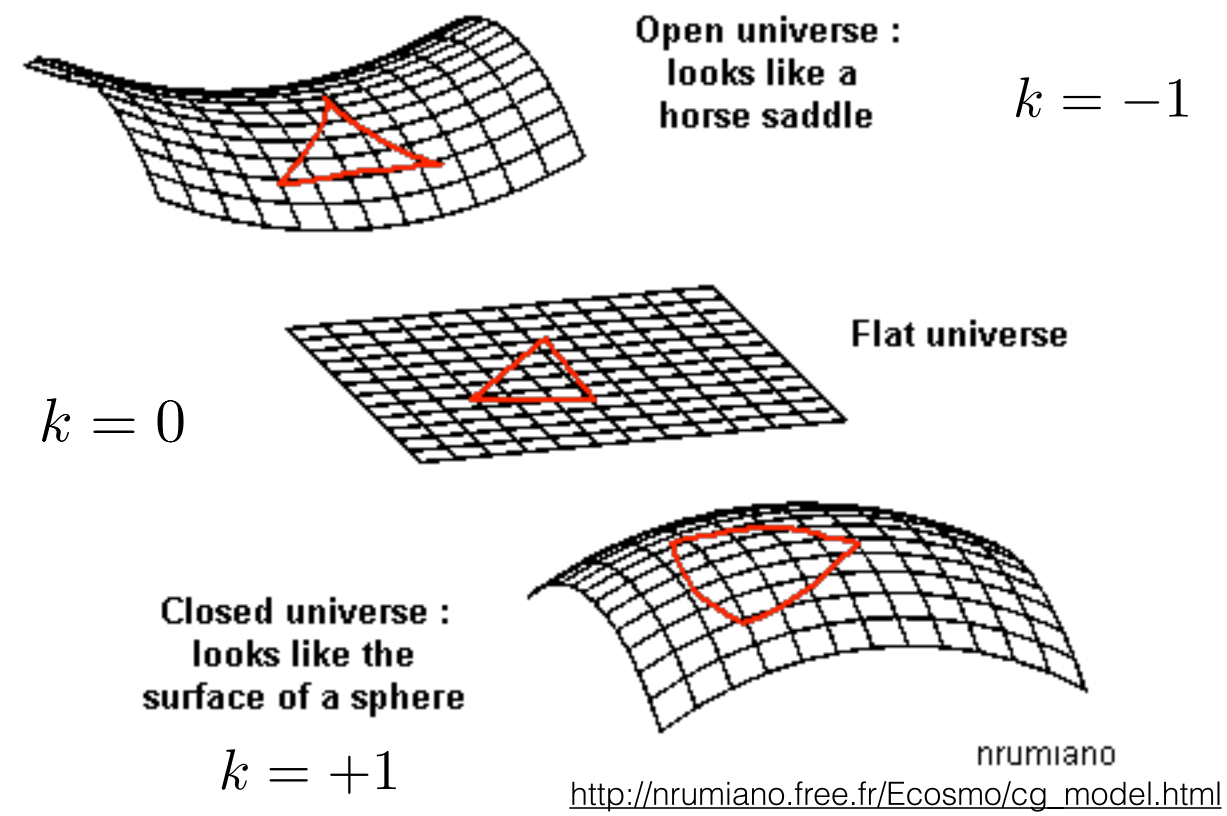


- Hamiltonian constraint incl. homog. field

$$C_H = p_\phi^2 - p_\alpha^2 - \cancel{4k\, e^{4\alpha}} + \cancel{4m^2\phi^2\, e^{6\alpha}} \approx 0$$

$$\alpha = \ln a$$

$\Rightarrow$ time evol. generated is gauge transf.

$$\dot\phi = \{\phi, C_H\} = 2p_\phi$$
$$\dot\alpha = \{\alpha, C_H\} = -2p_\alpha$$

set

$$\boxed{k = m = 0}$$

$\Rightarrow$ want to use α or ϕ as relational "clock" ← **both monotonic**

k=0 FRW with massless scalar

- Hamiltonian constraint of Klein-Gordon form

PH, arXiv:1811.00611

$$C_H = p_\phi^2 - p_\alpha^2 \approx 0$$

$$\Rightarrow \qquad \begin{aligned} \phi(t) &= 2p_\phi\, t + \phi_0 \\ \alpha(t) &= -\,2p_\alpha\, t + \alpha_0 \end{aligned}$$

- choose α as "clock" and affine evolving $\Phi = \phi\, p_\phi$

$$\Rightarrow \quad \Phi(\tau) = -p_\alpha(\tau - \alpha) + \Phi \qquad \text{rel. Dirac observable}$$

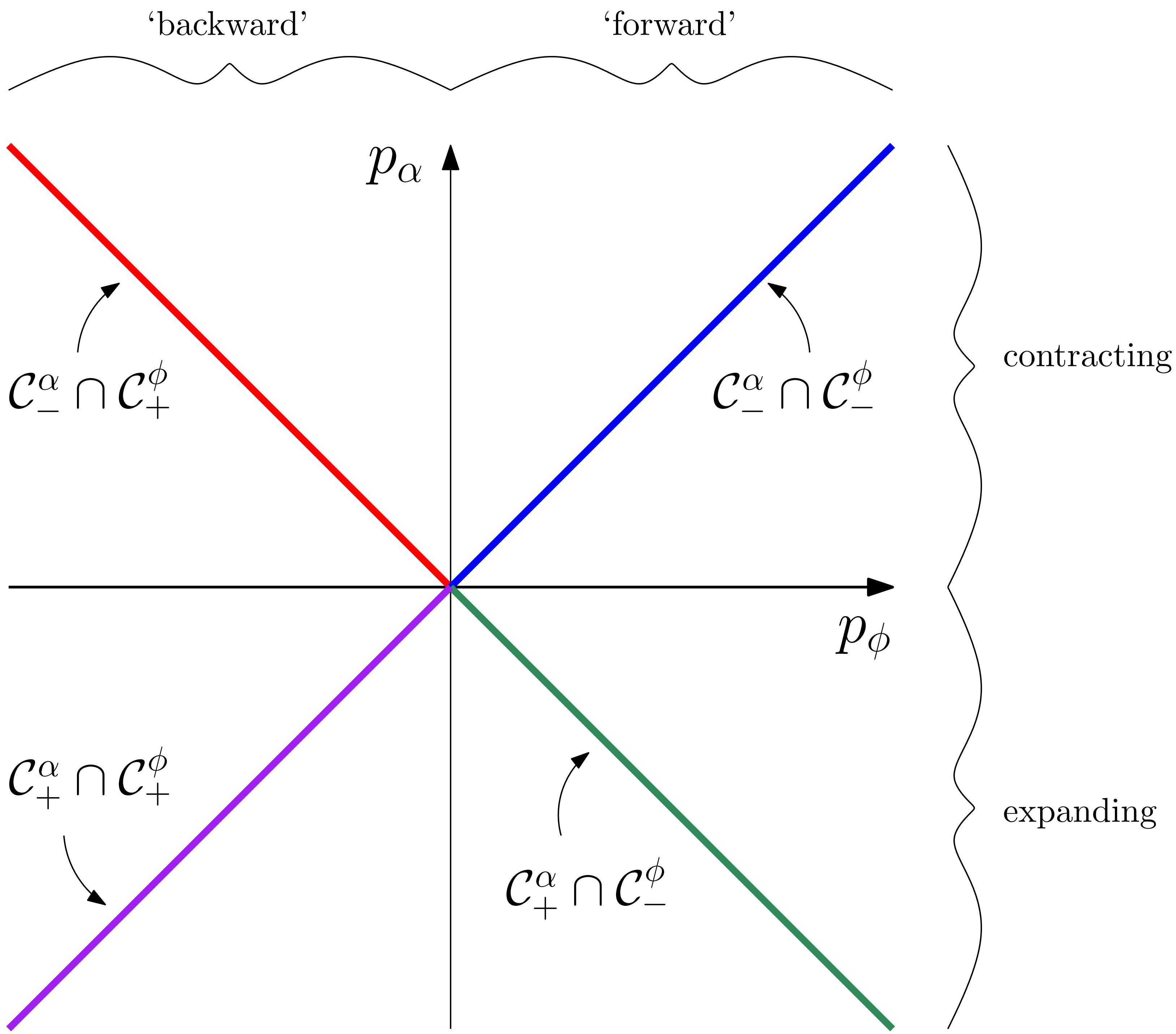
- get rid of redundant α, p_α

$$p_\alpha = \pm|p_\phi| = \pm H \qquad\qquad \Phi_\pm(\tau) = \pm|p_\phi|\tau + \Phi$$

generates fwd/bwd evol.

on reduced phase space

Positive and negative frequency sectors



Internal time switch in quantum cosmology

clock-neutral

PH, arXiv:1811.00611

$$
\begin{tikzcd}
 & \mathcal{H}_{\text{phys}} \arrow[dr, "\mathcal{T}_{\phi\pm}"] & \\
\mathcal{H}^{\phi(\alpha)}_{\text{phys}} \arrow[ur, "\mathcal{T}^{-1}_{\alpha+}"] & & \mathcal{H}^{\alpha(\phi)}_{\text{phys}} \arrow[d, "2\sqrt{2\pi}\;{}_{\phi}\langle\phi=0|\,\theta(\mp\hat{p}_\phi)\,\sqrt{|\hat{p}_\alpha|}"] \\
\mathcal{H}^{\phi(\alpha)}_{+} \arrow[u, "(\sqrt{\widehat{|p_\phi|}})^{-1}\,\frac{1}{2}\,|p_\alpha=-\epsilon\rangle_\alpha\otimes"] \arrow[rr, "\hat{\mathcal{S}}_{\alpha_+\to\phi\pm}"] & & \mathcal{H}^{\alpha(\phi)}_{\pm}
\end{tikzcd}
$$

evolution w.r.t. α

evolution w.r.t. ϕ

where

$$\mathcal{T}_\alpha = \mathcal{T}_{\alpha+} + \mathcal{T}_{\alpha-} \qquad \mathcal{T}_{\alpha\pm} = e^{\pm i\hat{\alpha}(|\hat{p}_\phi|-\epsilon)}\,\theta(\mp\hat{p}_\alpha)$$

works the same way

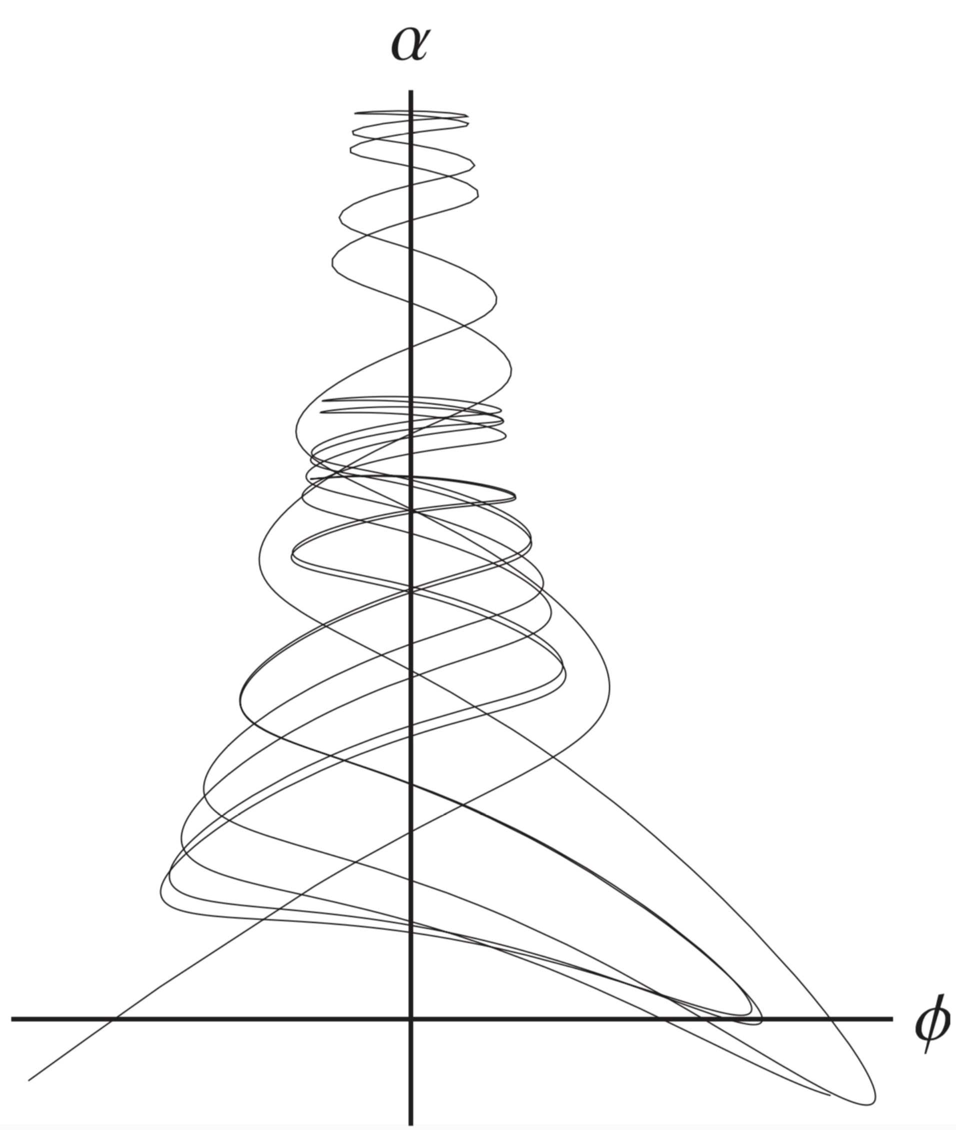
Relational dynamics more generally

- can get arbitrarily complicated

$$C_H = p_\phi^2 - p_\alpha^2 - 4k\, e^{4\alpha} + 4m^2\phi^2\, e^{6\alpha} \approx 0$$

chaos for massive field

interacting clocks,
global problem of time,
non-unitarity, transient clocks



see
Bojowald, PH, Tsobanjan, CQG 28,035006, (2011)
Bojowald, PH, Tsobanjan, PRD 83,125023 (2011)
PH, Kubalova, Tsobanjan, PRD 86, 065014 (2012)
Dittrich, PH, Koslowski, Nelson, PLB 769, 554 (2017)

New perspective on “wave function of the universe” PH arXiv:1811.00611

- no global operational state —> global state perspective neutral
- only relative operational states

“Wave function of the universe” as a perspective-neutral state

Operational interpretation from transformation to specific reduced theory

PH, Quantum 1, 38 (2017)
PH, JPCS 880, 012014 (2017)